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# SYSTEMATIC ANALYSIS OF SOURCE LOCALIZATION USING PE METHODS WITH BACK-PROPAGATION TECHNIQUES (U)

L. Nghiem-Phu, F.D. Tappert & S.C. Daubin
(Transcribed & Compiled by D.J.Thomson)

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Approved by
J.S. Kennedy
Head, Underwater Acoustics

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# Abstract

This document is a literal transcription of a video-taped talk presented by Fred Tappert during the Workshop on Acoustic Source Localization, 20–21 November 1985, held at the Naval Research Laboratory in Washington, D C. Questions and comments by participants are included Some equations were altered slightly to accommodate blackboard dynamics Several 35-mm slides that were projected at the end of the talk have been omitted from this transcription where indicated A list of contemporary references cited at appropriate places during the talk is included Finally, an extended summary that the authors provided to the organizers of the Workshop is included as an appendix



# Résumé

Le présent document est une transcription littérale d'un exposé enregistré sur bande magnétoscopique présenté par Fred Tappert lors de l'atelier sur la localisation des sources acoustiques qui s'est tenu les 20 et 21 novembre 1985 au Naval Research Laboratory à Washington D.C. Les questions et commentaires des participants y sont inclus Certaines équations ont été légèrement modifiées pour tenir compte de la dynamique du tableau noir. Plusieurs des diapositives de 35 mm projetées à la fin de l'exposé ne figurent pas dans le texte, là où il en est fait mention Une liste des références citées durant l'exposé est incluse. Enfin, un résumé additionnel fourni par les auteurs aux organisateurs figure en annexe

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**DREA TM 1999–059**

# Systematic Analysis of Source Localization Using PE Methods with Back-Propagation Techniques

by

**L. Nghiem-Phu, F.D. Tappert & S.C. Daubin**

**(Transcribed & Compiled by D.J. Thomson)**

## Executive Summary

### Introduction

Source localization techniques using model-based matched field processing (MFP) concepts have been developed in recent years for improving the capability of passive and active sonar systems. In 1985, a fast backpropagation MFP algorithm based on the parabolic equation (PE) model was presented at a Workshop on Acoustic Localization held at the Naval Research Laboratory in Washington, D.C. In order to record the underlying theory needed to implement and evaluate the method, this document contains a literal transcription of a video-taped version of the 1985 presentation and its subsequent integration with equations, slides and contemporary references.

### Principal Results

For an $N$-element vertical line array (VLA), the correlation surface spanning the target search domain can be generated $N$ times faster using the backpropagation method compared to conventional MFP methods. This speed gain could provide a potential advantage in any MFP system deployed for surveillance or rapid environmental assessment.

### Significance of the Results

The backpropagation MFP algorithm is compatible with DREA's ongoing efforts to improve its PE modelling capability. MFP source localization issues raised at the 1985 Workshop--normalization, environmental mismatch, sidelobe suppression—are still active areas of research. The speed gain combined with a PE model's range-dependent capability makes the proposed backpropagation method attractive for at-sea MFP systems.

### Future Plans

MFP methods can be combined with global optimization algorithms in order to invert array data for geoacoustic parameters of the sea-bottom—called focalization or matched field inversion. The fast method described herein could significantly reduce the time needed to obtain suitable inversions in this computationally demanding procedure. Incoherent and coherent frequency averaging for reducing sidelobes is currently being analyzed. The extension of the method to horizontal line arrays is being considered.

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# 1 Background

On November 20 and 21, 1985, the Naval Research Laboratory (NRL) sponsored a Workshop on Acoustic Source Localization, which was held at NRL, Washington, D.C. The aim of the workshop was to provide an opportunity for workers engaged in the area of research known as matched-field processing (MFP) to meet together and exchange results and ideas, both experimental and theoretical, in an informal context. Although a proceedings of the papers presented at this gathering was never published, an informal preliminary draft version was circulated to participants [1]. The concept of a workshop dedicated to the topic of matched-field processing in underwater acoustics proved to be so successful that subsequent MFP workshops were held every two years or so for several years, alternately at NRL and at the Defence Research Establishment Pacific (DREP) in Victoria, B.C.

A feature common to each of the NRL/DREP MFP workshops was that every presentation was recorded on video tape so that copies could be made and distributed to participants. This document contains the literal transcription of the talk given by Fred Tappert at the 1985 venue Tappert's presentation concerned "acoustic retrogation," a novel source localization scheme that uses a backpropagation algorithm based on the parabolic equation (PE). For an $N$-element vertical line array, this (unnormalized) processor provides a potential speed-up factor of $N$ when compared to the conventional (normalized) matched-field processor.

Although the backpropagation technique was introduced nearly fifteen years ago, almost no description of it has appeared in the open literature. Indeed, even in a book devoted to the subject of MFP for underwater acoustics [2], only a brief mention of the method is provided. Accordingly, in order to support DREA's ongoing MFP development, which uses a PE code for much of its modelling capability, it was decided to transcribe the video-tape recording of Tappert's presentation with a view to documenting the underlying theory. It soon became apparent, however, that a literal transcription of the complete talk would be helpful, not just for understanding the theoretical and computational details, but for recording Tappert's perspective on source localization. His viewpoint is that of the acoustic modeller (physicist), and not that of the signal processor, so he casts the source localization problem in terms of ideas and concepts familiar to wave propagation.

Although the transcription is literal, a few amendments were made. A bibliography of contemporary papers cited during the talk has been provided. Some viewgraphs that were projected at the end of the talk have been omitted where indicated. The slides were captured from the video signal using frame-grabbing software, then cropped and auto-balanced to enhance the contrast. The quality of the resulting grey-scale images is poor but reasonably faithful to the colour quality of the video tape used in the transcription. An extended summary that the authors' provided to the organizers of the workshop is appended with permission Finally, we remark that the "acoustic retrogation" concept described herein is closely related to some other PE-based work [3]–[7] and provided the motivation for our own analysis of the backpropagation algorithm for matched-field processing applications [8, 9].

## 2 Transcription

**Figure 1:** *Fred Tappert.*

RON DICUS: Fred Tappert, University of Miami.

I think I'll try a blackboard talk I feel much more comfortable coming from a University speaking in front of a blackboard. I do have a bunch of 35-mm slides that I'll show later.

I come at this, of course, from the point of view of the acoustic modeller. The background history is a little bit curious, and perhaps interesting. About four years ago, we in Miami—when I say we, it means my co-workers and I, not a royal we—and my two—the three of us, Dr. Lan Nghiem-Phu and Dr. Scott Daubin—the three of us have been working together for about five years and everything we do is in close collaboration, and the authors of the talk really are all three of us

About four years ago we got a contract to construct a special-purpose computer system to solve the parabolic wave equation acoustic model [10]–[12]. Those were the days when you could submit an unsolicited proposal and be awarded a contract without competition. The system we named PESOGEN—stands for Parabolic Equation SOlution GENerator—and the heart of the system is a microcomputer, a PDP–11/24. Built into that is an array processor, made by Cambridge Design Associates called the MSP–3000 rated at about 5 Megaflops The array processor is actually a pretty big box—together with the host and the graphics system you have a fairly large, substantial computer system—not very flexible because the whole system was really designed solely to compute solutions to the parabolic equation

What we compute you can think of as being a Green's function of the acoustic wave equation. The Green's function $g(\vec{x}, \vec{x'})$ is defined, say for the Helmholtz equation—in the simplest case—

$$\left[\nabla^2 + \frac{\omega^2}{c^2(\vec{x})}\right] g(\vec{x}, \vec{x'}) = -4\pi R_0 \delta^3(\vec{x} - \vec{x'})$$

—the point source referenced to one yard—a three-dimensional delta function In physics and mathematics, this [$g$] is called a Green's function. $R_0$ is the reference range 1 yard.

Of course, we don't actually solve this equation—we solve a more complicated one that has losses in it, has density in it. The model that we have also has a full geoacoustic bottom in it, rough bottom scattering, rough surface scattering, and the option for volume scattering. The scattering is done in a Monte-Carlo way so we actually include in the calculation the scattered energy, so we get spreads in angle, in frequency and in time. Now the software—right now, the model that we run is in software form—and we make a distinction between the software, PESOGEN being the hardware. In the future, we will integrate the software into the hardware and burn in PROM chips, and in that case, we can speak of the whole system as being a PE solution generator.

The software itself, we call it the Miami PE model, and it's different from other available PE models in many ways. Miami PE model—abbreviated MIPE—could be pronounced "MY" PE model [Laughter]. Our feeling is—as one of these unprovable things—that MIPE model actually is able to compute the correct Green's function of the real ocean That's an enormous claim. Of course it's not true uniformly in frequency and range and environmental conditions, but by and large, we feel that it is sufficiently good for applications—sufficiently accurate and reliable and it's very, very fast.

Having done this—the work was completed about two years ago—we thought there would be a lot of Navy people coming to us and asking to use one or borrow one or buy one. We had our order books out and ready to sign but no customers. The reason for that—I think you know that the acoustic modelling business is very competitive. People always claim that their own model is better than anyone else's. There are certain projects that do require fast accurate computation of acoustic Green's functions but there seems to be a—I wouldn't call it a conspiracy but the people that are doing the work now insist on continuing doing the work The models that are used in practical performance prediction are RAYMODE, FACT, ASTRAL and those people have no intention of using the PE model. At least we couldn't convince them of that So, we looked around.

> OREST DIACHOK: They are interested in using their own PE model, not your PE model

Not even their own, very often. Here at NRL you do have a pretty good PE model, but it doesn't have all the physics and the special features of "MY" PE model [Laughter]. So, we were in the classical position of having a solution to some problem, but not knowing what the problem is. We couldn't find a problem where our solution—the PE solution generator—was desired. So, we spent a lot of time trying to think of problems. The Navy R & D system doesn't seem to be very good at matching up solutions with problems. We have a solution to many Navy type problems like localization but no one came to us and said "Here's a problem Can you solve it?" We had to go looking for our own problem. We got together with a very knowledgeable person, from a company—System Planning Corporation—which is here in the D.C. area, and tried to brainstorm the application area assuming that we did have a very good Green's function for the real ocean That person—Bruce Wooden—maybe some of you know him—is very knowledgeable about all areas of high-tech knowledge relating to military applications but he doesn't know anything about underwater acoustic modelling. But he, in his naivity, was ready to believe that we could produce the true Green's function of the ocean What could you do with that?

Without too much thought, if you accept that hypothesis that we are able to predict—to compute accurately and reliably the Green's function of the ocean—there's all kinds of Navy ASW problems that can be solved immediately. Clarence [Clay] has spoke [*sic*] about measuring the sound pressure level at a single receiver as a function of time. You get a very complicated transmission loss curve. If you were able to predict accurately, in detail, the transmission loss curve, then, in principle, you could just do a pattern matching or pattern recognition or replica-type processing and determine immediately the track of the target. Also, in active sonar, [there are] a lot of applications for Green's functions, in localizing the target, measuring the true target—true target strength referenced to 1 yard. [In] passive narrowband sonars, there is a lot of intelligence interest in measuring true source levels. If you make a measurement at a range, you want to get it back to reference range 1 yard, get the true source level You need a very accurate backpropagation model—or forward—it doesn't really matter. It doesn't matter because reciprocity says that these Green's functions are equal if you interchange source and receiver—

$$g(\vec{x}, \vec{x'}) = g(\vec{x'}, \vec{x}).$$

Here [in $\delta^3(\vec{x} - \vec{x'})$] you think of $\vec{x'}$ being the source and you compute the field at a point $\vec{x}$. If you interchange source and receiver, the Green's function is reciprocal—on the basis of that equation and also on the basis of other, more complicated, more realistic equations, such as the parabolic equation It turns out that the parabolic equation is exactly reciprocal. The approximation does not violate reciprocity.

In other applications we looked at, we finally hit on this idea of matched—equivalent to matched-field processing for localization We first went at it from the point of view of locating a transient from an unknown target That's because we're studying the properties of MIPE model for predicting the pulse response function due to a bandlimited type source [13] We showed that the pulse response functions are reciprocal within our model and therefore, in principle, if you really believe that prediction—if it is the true Green's function—the true pulse response function—all you have to do is measure the pulse response function—measure the transient—and then by backpropagating at some unknown location, that transient will collapse into a very narrow pulse. The pulse spreads as it goes out from the source because of multipath effects

> ROBERT KOCH The reciprocity relation says that you have to substitute the source at the receiver location. It doesn't mean that you can take the received signal and propagate it to get back to the source. It's a different sort of problem

It is a different sort of problem but reciprocity holds frequency by frequency and from that it follows that the pulse waveform is also reciprocal. I published an article on this [14]—

> KOCH I agree that if you take one waveform and propagate it this way, then you get the same result if you take that waveform and propagate it backward, but I don't think that you can propagate the waveform over here and send it back and say that's reciprocity
>
> CLARENCE CLAY: There's a neat little experiment that I was involved in many years ago We transmitted an impulse from one place and got a waveform over

here and then we time-reversed it and transmitted it and got back the correlation function—

It turns out that it's an old idea, Clarence and Parvulescu are known as the creators of that concept [15]. Only fairly recently, in fact at the beginning of this year, did we get into CW localization—single frequency or tonal type sources—and I didn't learn about all this other ongoing work until April of this year [1985] at the Acoustical Society meeting in Austin. That's when I learned about Homer's [Bucker] work [16] and yours, Art [Baggeroer]—I'm not sure when I learned about that. You [Clay] sent me a reprint of your 1966 paper [17] many years ago in connection with coupled mode type modeling and I think I skipped through the beginning of it, and went to the coupled modes—that's what I was most interested in. So I apologize for all the work at NRL here [that] is unknown to me, or at least as far as I'm concerned—

TAPE CHANGE—

After thinking about this class of applications, we—together with some people from SPC—came up with a rather broad, general concept, which is that, if you can accurately and reliably predict acoustic Green's functions for the ocean, then you can take any measured signal and effectively deconvolve with the computed Green's function, and in that way, remove completely the influence of the ocean. All the environmental influences of the ocean on sound propagation are in this Green's function. If you can predict that, then you can deconvolve or take it out of the received signal. By doing that, you get, in effect, the signals back at reference range 1 yard from the source.

Now the broad concept works for narrowband passive, broadband passive, active, and, in some sense, it means you can do—you can make any sonar system perform enormously better than any currently existing sonar system—localization, detection, classification, tracking—in a sense, it's the ultimate solution to the ASW problem. This summer I submitted an abstract to the Acoustical Society meeting [Laughter]—the title was: "How to develop the ultimate ASW weapon." Bill [Kuperman], and Art here looked at that and said: "At a professional society meeting we don't talk about weapons, so let's change the title." It was watered down, but I—I still feel that it could be—this basic concept could be developed into a very powerful sonar system—of revolutionary importance.

Now the gut issue, then, is whether or not you can accurately predict acoustic Green's functions. I think every reasonable person would have to admit that, if you think about it, if you could do this, if you could compute these Green's functions, then you could do an awful lot in ASW that cannot be done now. So computing Green's functions is what I think I know something about. I've been doing it for years. I think, at Miami, we are pretty good at computing Green's functions. We get good accuracy, reliability—meaning uniform accuracy in all reasonable environments—and we do this very fast.

Now if you—so the first statement is an if statement, an hypothesis, if you can do it. The second statement is we believe that we actually can do it. Now that could be viewed as a threatening-type situation. Navy people interested in threat assessment—if we can do it, so can the other side— the Communist Block. If they develop this capability, then the security of our FBM [Fleet Ballistic Missle] forces will be threatened—in a very serious way. So I take this whole matter of computing Green's functions very seriously—we do, at Miami. I think

other people do too—it's not this free-wheeling basic research. If we can do what we say—as an acoustic modeller, I think it—I won't go too far in saying we can do it right now—but certainly it is possible, without a major effort—a major increase in funding—to improve the PE model to the extent that it really can make accurate predictions of the Green's functions. Once you have that capability, you have a—not the solution to the ASW problem—but, in a certain sense you do—you can locate, you can detect, you can classify submarines anywhere in the world's oceans—even against very low threat source levels.

That was the motivation for this, finally, and I think what I'd like to do is to go through some equations here which reflect the way that I learned to think about localization for matched-field type processing. I honestly know very little about signal processing. I've tried to learn it, but when I read articles in books I don't see anything about the ocean, I don't see anything about the acoustic wave equation—with some exceptions. The most notable exception is the review article written by Art about 10 years ago [18]. In the book, it even had a sound speed profile, it had ray trace plots, and I think the acoustic wave equation But that's very exceptional. Hardly anyone in signal processing even writes down the wave equation.

> T.C. YANG. I would like to inject a question and see how you respond to that. Suppose I just give you a very simple assumption—that it takes 10 modes to produce an impulse—an impulse equivalent to 10 modes. Ok, can you propagate to the receiver? Due to transmission loss or anything you only receive 4 dominant modes The other 6 modes are too small in signal-to-noise ratio—they become noise, you don't measure it. Now you take these 4 modes you receive, deconvolve with whatever Green's function you have, you can never produce the 10 modes you start with, and you never produce the original impulse. So, whether you know the Green's function that way or not, [it] is not the solution you get

There certainly is a limitation—acoustic fields in the ocean—no sensible person would claim that they're totally predictable There are unknown fluctuations in the ocean, turbulence or internal waves, or say the sea state The sea state—the statistics might be predictable, but the actual displacement of the surface, at one instant of time, is not predictable As a consequence, the acoustic fields, depending on frequency and range, have a component which is unpredictable, now —

> BILL KUPERMAN I'd like to ask a question, the question is about a signal that is masked by the noise, isn't that what you're saying?
>
> YANG Some information is lost.
>
> KUPERMAN: Yeah, because if there's noise, I mean it's there, if the noise in that same region is bigger than the signal—

It's a very similar problem—if you lose coherence or if you lose energy, you cannot reconstruct—you cannot get back to the source. There is a limitation of range there is a limitation of frequency But I think the regime in range and frequency is large enough to be very interesting in applications If you try to locate a target 10,000 miles away—you can't do it

for many reasons—but one is that you will never be able to do predictions of the Green's function for 10,000 miles. So, certainly there is [*sic*] limitations, and it's important to find out what they are

I'll try to convey to you some of what I know about the limitations of this type of signal processing. It should have a name—Homer calls it matched-field method [16], Clarence calls it matched modes [19]–[21], NRL simply call it ambiguity processing [22]–[24]—

CLAY: I call it array matched filtering.

—matched filtering [17, 25]. There is another kind of signal processing called adaptive signal processing where you try to be as independent of the environment as you possibly can. This is just the opposite, of course—you make use of every bit of environmental information you can and compute these Green's functions—so it's totally dependent on your knowledge, on your ability to predict the Green's function. So it might be called predictive processing, as opposed to say adaptive processing. It's the ability to predict Green's functions which is the critical element of this kind of processing. Whether you match or not—now these maximum likelihood type methods [26, 27], it's hard to interpret that as a match, although I suppose you could It's not a simple correlation. Many of the things that we actually compute don't look like any matching process. It all works out to the same thing in the end—the ability to localize based on computed Green's functions.

We started out thinking about reciprocity—if you had $N$ sensors, located at positions $\vec{x}_n$ $n = 1, 2, \ldots N$. In other words, we have assumed that these sensor positions are known. If they're not known, Homer's ideas [28, 29] could be very successful. The pressure at each of these sensors, in the ideal case, are obtained ideally without noise, obtained from an FFT-type of processing, within one frequency bin, if there's a tonal there, the pressure would be $p_n$ a complex quantity, real and imaginary parts or quadrature components, measured in many ways. In terms of the Green's function, this is given by

$$p_n = g(\vec{x}_n, \vec{x}_s)p_{s0}$$

for the $n$th receiver $\vec{x}_n$ and the source position $\vec{x}_s$ times $p_{s0}$—$p_{s0}$ is the pressure referenced to 1 yard The source level is directly related to the magnitude of $p_{s0}$. The Green's function is the acoustic field relative to 1 yard from the source location to the receiver location, and that gives you the pressure at the $n$th receiver.

Now you solve for—of course $p_{s0}$ is unknown—so you might think of solving for $p_{s0}$. That's basically a deconvolution process—you take your received pressures—divide by the Green's function—and let me introduce a notation here—$g_n(\vec{x}_s)$—let me add another equation here. By reciprocity, this is also

$$p_n = g(\vec{x}_n, \vec{x}_s)p_{s0} = g(\vec{x}_s, \vec{x}_n)p_{s0}$$

which means now that we propagate from the $n$th receiver to the source. You go from the second argument to the first The receiver positions are known, so we can just introduce the notation that $g_n(\vec{x}_s) = g(\vec{x}_s, \vec{x}_n)$, the other way [to obtain]

$$p_{s0} = \frac{p_n}{g_n(\vec{x}_s)}.$$

This is what our PESOGEN computes—it computes the Green's function $[g_n(\vec{x}_s)]$ out from the $n$th receiver to the source location This is true for all $n$. So you can sum from $n$ equal 1 to $N$—$p_n/g_n(\vec{x}_s)$—and divide by $1/N$—

$$p_{s0} = \frac{1}{N}\sum_{n=1}^{N}\frac{p_n}{g_n(\vec{x}_s)}$$

—a very simple identity Now, that leads you to consider—since you don't know really the source $\vec{x}_s$—you can consider a hypothetical field—the field is just any function of position—defined by

$$h(\vec{x}) = \frac{1}{N}\sum_{n=1}^{N}\frac{p_n}{g_n(\vec{x})}$$

—$\vec{x}$ is the unknown This is what we compute. The $p_n$'s are a set of measured pressures The hypothetical field $h(\vec{x})$ is something that is computable. Now, it has the property, that if it s evaluated at the actual source location, it gives the 1-yard pressure—the source level. $h(\vec{x}_s) = p_{s0}$. Beside $h$, we also compute what Homer calls a detection function, $F(\vec{x}) = |h(\vec{x})|^2$ By construction. of course, that's equal to the square of the 1-yard pressure at the source So we get a source level—let's call it source level primed—because it is not a true source level—is a function of the unknown location $\vec{x}$—write that—define it to be 10 log the ratio of $F$—which is $|h|^2$—divided by the square of the reference pressure.

$$\mathrm{SL}'(\vec{x}) = 10\log\left(\frac{F(\vec{x})}{p_r^2}\right).$$

The reference pressure is 1 $\mu$Pa Now. by construction, you see if you evaluate this at the actual source location. $\vec{x}_s$. you get 10 log the ratio of the 1-yard pressure squared to the reference pressure squared.

$$\mathrm{SL}'(\vec{x}_s) = 10\log\left(\frac{|p_{s0}|^2}{p_r^2}\right) = \mathrm{SL}.$$

which is by definition the source level in units of dB [re 1] $\mu$Pa

Well that's all fine if you can figure out where $\vec{x}_s$ is in this field of unknown locations What you ve been able to do is to propagate back to 1 yard and get the source level Now why—in this particular function $[\sum_{n=1}^{N} p_n/g_n(\vec{x})]$—by construction. when $\vec{x}$ is equal to $\vec{x}_s$. all the terms in the sum are exactly equal—they add up coherently, and they give the value $p_{s0}$ At other locations—$\vec{x}$ not equal to the source location—you really have no idea what this is Reciprocity only tells you what happens at these source locations. [For] some other location reciprocity says nothing about the value of this function However in the real ocean. the Green's function is a very complicated function of position. and you'd expect. on the average. very little coherence or correlation between the numerator and denominator. so this will add up incoherently If you square it $[F(\vec{x}) = |h(\vec{x})|^2]$—it's a double sum—you add it up incoherently and. on the average. you'd expect that if $\vec{x}$ is—I'll say not equal [to $\vec{x}_s$] but —of course if I come to it. Green s functions are continuous functions. so very close to the actual source it's going to behave smoothly—but if you get a few wavelengths away—by

and large, and not try and sharpen up that estimate, you get an incoherent sum, so if $\vec{x} \neq \vec{x}_s$, $F(\vec{x})$, in order of magnitude, would be—

$$F(\vec{x}) \sim \frac{|p_{s0}|^2}{N},$$

—a very intuitive notion Several people here alluded to the observation from numerical simulations that it seems to be true. What it amounts to is getting—this is down by $1/N$ compared to the value at the actual source since by construction $F(\vec{x}_s) = |p_{s0}|^2$. You have $1/N$—that amounts to an array gain of $10 \log N$ and to order of magnitude that's true. We have found that in our simulations, as long as the $N$ sensors are independent. If you put them all together at one point, of course, you don't get $1/N$. In fact you get the same—the full value of $p_{s0}$

This is one way to do it. Now, it's not the best way, in fact, I don't think [it's] very successful It's very likely that this is identical [to]—within a factor—the same as the maximum likelihood method We had a great difficulty with this, for the following reason. It's obvious—the Green's function is in the denominator. Green's functions in the real ocean have many, many zeros. They're all over the place. The zeros arise from multipath interference If you have a single source in an infinite uniform ocean, there are no zeros in the Green's function. But as soon as you get boundaries or sound speed profiles giving rise to multipath, then you have zeros.

KOCH: You are working in frequency space here?

This is all frequency space. I can say more about that later, but this is frequency—[For a] single frequency, if you look at any full-wave model output such as PE, you find zeros all over the place It's really true That makes—it means that $h(\vec{x})$ has infinite values or arbitrarily large values depending on the numerical mesh that you use to compute the Green's function. Mel Pedersen calls these zeros amphidromic points.[1] If you look at the phase of the Green's function in the neighbourhood of the zero, you find that it—if you represent $g(\vec{x})$ as an amplitude and phase— $g(\vec{x}) = A(\vec{x}) \exp i\phi(\vec{x})$—phase can only be defined modulo $2\pi$. If you look at contours of $\phi = \text{const.}$, say in a plane, if you have a zero here, then these equal phase lines look like that If you have another zero, say down here, then the equal phase lines kind of look like that. Very complicated and interesting structure.

More than ten years ago, Ron Hardin and I, when I was at Bell Labs, made plots of PE model output—surfaces of constant phase. That's what it looks like. They're the real things. It's not a matter of some kind of approximation. The more accurate the Green's function is, the deeper the nulls will be And I think that it makes this particular—what that means is that when you search over here—on the average $F$ will be—on the average—be down by $1/N$. But there will be points where $F$ is very high. However, the basic intuitive notion still holds—that at the source location you do get coherent addition and there will be a sharp local maximum. The value at the maximum is the 1-yard pressure. Unfortunately, there may be—and we find there usually is, other maxima They are not target locations. Now, you can avoid that by adding to the Green's function white noise Prewhitening is a very

[1] An amphidromic point is a point in the ocean at which there is no tide and around which the cotidal lines rotate over a tidal cycle

**Figure 2:** *Sketch of amphidromic points.*

typical signal processing technique. I can't bring myself to add noise to a computed Green's function after spending so much effort trying to get it as accurate as we can [Laughter]. I am not going to turn around and add noise to it. So now we have to learn to live with these false targets—or come up with a different algorithm.

There are many similar algorithms, by no means unique. Another one, instead of solving for $p_{s0}$, solve for $1/p_{s0}$. This is equal to the Green's function divided by $p_n$, but that's independent of $n$, so we can just sum from $n$ equal to 1 to $N$ the inverse—$g_n(\vec{x}_s)$ divided by $p_n$.

$$\frac{1}{p_{s0}} = \frac{1}{N}\sum_{n=1}^{N}\frac{g_n(\vec{x}_s)}{p_n}$$

The hypothetical field in this case—it's a different $h$—a different hypothetical field—the same symbol—replace the unknown $\vec{x}_s$ by $\vec{x}$. You just compute a quantity like this, with the Green's function in the numerator.

$$h(\vec{x}) = \frac{1}{N}\sum_{n=1}^{N}\frac{g_n(\vec{x})}{p_n}.$$

Now you can't do any damage—the zeros of the Green's function can't do any damage up in the numerator, but the received pressures may be very small if one of the sensors happens to be near a zero of the acoustic field, or say, if it's in a shadow zone or a region of very high loss. If you simply implement this you have the same kind of problems, in simulations.

> DAVE BERMAN: When you have also competing nulls. I mean, in this case you are looking for a null, right, where the source is, there should be a deep null—

No, where the source is, then all these terms are equal and you get perfect coherent addition. It's away from the source—

> BERMAN: No, but the pressure at the source location should be big—$p_{s0}$ should be big—so $1/p_{s0}$ should be small.

No, it doesn't—it doesn't really work like that. Where $\vec{x}$ is equal to $\vec{x}_s$, then each of these terms [in the sum] is exactly equal to $1/p_{s0}$, so you get perfect coherent addition. At other locations that are sufficiently far enough away from the source location, this sum will add up incoherently. Once again you get a background—we call this background—local—what could we call it—we call it localization noise. There is no noise in this, no true ambient noise or self noise. But when you do this type of localization there is always a background. It is not zero. The background in general, or on the average, is down by $1/N$—when you square the sum—you get an incoherent addition. But the pressures [$p_n$] themselves can be small—and that—you are weighting the hydrophones inversely to the actual pressures there. It doesn't seem very—like a good idea if one of the hydrophones happens to be in a shadow zone. You're trying to increase its weight as much as you can to compensate for the small value of the Green's function at the source location. Here [in the denominator] you can add some noise. If there is noise, what happens here is that it actually gets better—it stabilizes the computation—a little bit of noise. But if you add a lot of noise then you can't match very well and performance is poor. So I don't really like this method either.

Now the two methods—first we started with $p_{s0}$—then the reciprocal—you can choose any other function of $p_{s0}$—take a sum, or take a difference of these two, or you can take something like $\log p_{s0}$. You write it as a sum, as I did over here. You make a hypothetical function of the unknown position, and then by construction at the target it's going to have a local maximum or minimum Well the point is that there are an infinite number of formulas like this which, in principle, work perfectly and do the simulation with the computed Green's function—simulate the data using the same computation that you use for inversion—it does work You get exactly to the target location—you get exactly the source level. But it's not very stable—it's not very robust. Among this infinite class of functions, I as an acoustician can't choose between them Maybe signal processors have a way to do that. Even this class of functions does not exhaust the possible set of formulas. Another—of course, because we haven't come to the common correlation method yet.

Another class of formulas are based on weighting the different hydrophones. Introduce weights for the $n$ hydrophones—call them $w_n$. They don't even have to be real-valued or non-negative just normalized so that the values of the weights divided by $N$ add up to 1.

$$\frac{1}{N}\sum_{n=1}^{N} w_n = 1.$$

In the previous formulas in effect I chose all the weights to unity—an unshaded type of array. With this, let's go back to this formula here [$1/p_{s0} = \sum_{n=1}^{N} g_n(\vec{x}_s)/p_n$]. This ratio is just—it seems like a trivial identity—I know it is—you write that as

$$\frac{1}{p_{s0}} = \frac{1}{N}\sum_{n=1}^{N} \frac{w_n}{p_{s0}}.$$

Now we know that $p_{s0}$ itself is any one of these terms, such as $g_n(\vec{x}_s)/p_n$, so we also have the identity

$$\frac{1}{p_{s0}} = \frac{1}{N}\sum_{n=1}^{N} \frac{w_n}{p_{s0}} = \frac{1}{N}\sum_{n=1}^{N} \frac{w_n g_n(\vec{x}_s)}{p_n}$$

Now you want a hypothetical function—you replace the unknown $\vec{x}_s$ by $\vec{x}$ and you have a function $h(\vec{x})$

$$h(\vec{x}) = \frac{1}{N}\sum_{n=1}^{N}\frac{w_n g_n(\vec{x})}{p_n}.$$

Here's another whole class of functions. Now, how do you choose the weights?

One way—in this formula—you'd like to choose the weights to cancel out the possibility of small denominators. You have small denominators, it's not very good computationally, so you choose $w_n$ to be, say, the squared magnitude of $p_n$,

$$w_n = |p_n|^2.$$

Now, you want to normalize like this $[(1/N)\sum_{n=1}^{N} w_n = 1]$, so divide by $1/N$—sum 1 to $N$—of $|p_n|^2$.

$$w_n = \frac{|p_n|^2}{\frac{1}{N}\sum_{n=1}^{N}|p_n|^2}.$$

Now we have properly normalized weights. You put that in this formula—[points to formula for $h(\vec{x})$ above]—[mod] $p_n$ [squared] is equal to complex conjugate times $p_n$ so, in that case, the $1/N$'s cancel out—what you end up with is—sum 1 to $N$—$|p_n|$ squared times sum from 1 to capital $N$—$p_n$ complex conjugate—$g_n(\vec{x})$,

$$h(\vec{r}) = \frac{1}{N}\sum_{n=1}^{N}\frac{w_n g_n(\vec{r})}{p_n} = \frac{\sum_{n=1}^{N} p_n^* g_n(\vec{x})}{\left(\sum_{n=1}^{N}|p_n|^2\right)}.$$

Here's the correlation function—with an unusual normalization. This is the method that we normally use in Miami. We call this Method A.2 [Laughter] I try to classify all these different methods. We must have tried out—implemented—have working in our PESOGEN system—at least a dozen different methods. This is one It works—I'll show you results using this method—it works very well—especially for a large number of sensors In fact, it's almost indistinguishable from the normalized correlation function After you get this $h(\vec{r})$, of course, it still has the property like over here $[F(\vec{x}) = |p_{s0}|^2/N]$, if you evaluate it at the source location—if $g_n$ is then $1/p_n$ and the numerator is equal to the denominator except for $1/p_{s0}$ and you evaluate—you take the log of the square of $h$ and you divide it by your reference pressure, and when you get to the—when you discover the source location, you also have the source level, on your [unintelligible] way. And this method like the other method where I began with $1/p_{s0}$, has a very interesting property Since you have the Green's function in the numerator—Green's function satisfies the Helmholtz equation—you're taking really a linear combination of Green's functions. Therefore, this hypothetical field satisfies the wave equation—with this Green's function. Whatever wave equation it is that you're using $h(\vec{r})$ will satisfy the wave equation So if you happen to be using this very idealized wave equation --$h(\vec{r})$ satisfies—if you put in a $4\pi$ reference range—this is just a constant $[\sum_{n=1}^{N}|p_n|^2]$—the mean squared pressure averaged over all the receivers, so it's like a received sound pressure level Let's put that—let's call it $A = \sum_{n=1}^{N}|p_n|^2$. And then we have the sum over all the receiver locations—complex conjugate—complex conjugate is basically what Clarence was saying if you want to backpropagate using reciprocity you need to start out

with the complex conjugate of the fields. Then you have your point source—a collection of point sources located at the receiver locations which are assumed to be known—

$$\left[\nabla^2 + \frac{\omega^2}{c^2(\vec{x})}\right] h(\vec{x}) = -\frac{4\pi R_0}{A} \sum_{n=1}^{N} p_n^* \delta^3(\vec{x} - \vec{x_n}).$$

That means you can compute $h(\vec{x})$ in a single run of your Green's function machine or your Green's function program—acoustic model, whatever you are using—just one run, using linearity of the—linear superposition of the wave equation. By doing that you save a factor of $N$ in the computation compared to the other normalized correlation function method. The normalized correlation function method, in this notation, just for comparison or reference—call it normalized. I know there're elegant ways with vectors and matrices—the way I would write this—and you divide by the square root of the sum of the squares—$p_n$ squared times sum $g_n(\vec{r})$ squared—I think the $1/N$ still cancels out—

$$h_n(\vec{x}) = \frac{\sum_{n=1}^{N} p_n^* g_n(\vec{x})}{\sqrt{\left(\sum_{n=1}^{N} |p_n|^2\right)\left(\sum_{n=1}^{N} |g_n(\vec{x})|^2\right)}}.$$

This is the quantity that other people compute [in the] matched field method. We also compute it for comparison to, say, this function here [viz., the unnormalized correlation]. Computationally, the difference is that this function in the denominator $[\sum_{n=1}^{N} |g_n(\vec{x})|^2]$ is a variable. In order to compute this denominator, you have to calculate or compute separately each of the individual Green's functions. So this normalized correlation function you have to do $N$ computations. Whereas, with the unnormalized one, you saved a factor $N$. If you have a 100 sensors, you saved a clean factor of 100. So a typical computation of ours is always under a minute—I believe 10 or 20 seconds—deep ocean, 100 Hz, ranges of several hundred miles. So, let's say 1 minute for us, using this formula. Now, on the same machine, the same PESOGEN system, highly optimized, fully vectorized code, it would be 100 minutes—1 and 2/3 hours—with the other formula. We're talking about the difference between 1 minute and 2 hours or so. Also, with this [normalized] method, you have to store the solution, all the solutions, all your Green's functions have to be stored so you can add them up later. With this [unnormalized] method, you just compute in place—no need to store the solution—the PE method as you know—you march out from the sources—here the sources are actually the receivers - you march out from the receivers, you compute the fields $h(\vec{x})$ without ever storing it and you square them, log it, compute this SL' as a function of $\vec{x}$ and display. It's actually your display device that stores the computed fields.

> KOCH: On the other hand, if you change the set of $p_n$'s, for example, if you sample from time to time you have to recompute the field every time that you cut a new sample $p_n$ whereas if you store the $g_n$'s once then you can use them over and over again.

That's very true. The ocean—if you're absolutely sure the ocean is not changing during your measurement—that can be done.

> KOCH: Or if you don't know how it's changing

Or if you don't know how it's changing—sure. If you do know how it's changing—if the changes are predictable, then I would re-compute the Green's function.

UNKNOWN PARTICIPANT. How long does it take to compute the Green's function?

As I said—10 or 20 seconds Less, less actually—that's throughput time. There's very little I/O in this. except to the display And it's a dedicated machine, no one else is using it. So. from the time you say go, start computing. 10 or 20 seconds later you see the whole Green's function displayed out in the field. Well, in this case, what you—you see. here you're not computing a—you're computing $h$—you see the whole field $h$ displayed on the screen in dB units

KUPERMAN This $h$ you see is for one, for your source in one place. is that right? It's the ambiguity function for the source placed in one place

It turns out that it works also for many sources—

KUPERMAN No. I didn't mean that—I meant—I can understand how it could track in But now. 5 minutes later your device is somewhere else So you get a new $p_n$--that you measure—it's new Don't you have—you have to do this equation again—this whole business again—for the new $p_n$?

If you don't store it Yes, you have to do it again

KUPERMAN Well. I mean you have a new measurement, right? You—

Yes—even if you store it—well if you store—

KUPERMAN There's nothing to store, right. you don't separate—

If you store you can bring it out of memory and multiply and add

KUPERMAN As I understand it you're not calculating $g_n$—you are calculating $p_n q_n$ right?

KOCH You are calculating the sum of—

Yes. we're calculating the sum—

KUPERMAN The final result. Now you get another—make another measurement $p_n$ right. because the thing has moved

Yes the final result is different and we would re-compute Yes, we would not—store There's trade-offs The way we re thinking now—of course, we're very far from having anything operational in the way of a sonar system. Our thoughts about it are that we would simply re-compute As long as you're searching individual bearings Now if you're going to search in 3-dimensional space then your—the amount of computation goes up very rapidly

KUPERMAN: Is it true though to say that the difference is that in one case you keep a storage set of replica functions that doesn't change and you beat it against your measurements. In this case, since you are combining your replica function with the measurements you have to re-compute every time you have a new set of numbers at your hydrophones?

Yes If you get a new set of numbers every 10 seconds—Homer's system might work—you have to re-compute every 10 seconds. But we can do that already with a very simple little cheap machine.

KOCH. I think it would make more sense here if you use the fast computation that you've got to interactively vary the environmental parameters to make sure that you can't get a better match and store those numbers for $g_n$ for—in one of those—in doing this process over a number of observations.

ART BAGGEROER: Bill and I were having an argument while you were talking back here and—but now, as I interpret it, you are actually—this is regarding how you did the backpropagation—you are actually—that's a backpropagated field necessary to some of the migration algorithms. Is that correct?

One could think of it like that—[Competing voices]—It's a big complicated literature dealing with this type of—it's the inverse source problem. Holographers, in acoustics and optics do things like this all the time. Seismologists do it—exploratory seismologists—trying to image the subbottom layers Here we are trying to image a single omnidirectional source which can be modelled as a point source.

BAGGEROER. Now the issue I brought up is how you are computing that—you are actually computing by backpropagation.

We are actually literally backpropagating from receivers to unknown target locations.

BERMAN: This is the length of the array. If the array doesn't sample the whole water column for a line array for a small part of the water column—if you had a point source—if you had an infinite array—infinitely dense—obviously you would come right back on the source. If you had a finite array, sparse for example—

Let me talk about that—

KUPERMAN. One other question before that. Say we have a range-independent environment and let's say that the perfect ocean that we're talking about didn't have all of the complications that you have in your PE, and—the simplest case—a range-independent, deterministic ocean—then one could take this equation that you wrote down here on the right and just solve it by any other method. The method that you're talking about—you know, in a range-independent environment very often other methods are faster than the PE and you get the field all over where you want it, so in that case, in a range-independent environment, you just solve it any way anybody wants to solve the wave equation—that's the wave equation

**Figure 3:** *Sketch of lens resolution analogy.*

Yes. When we spoke about resolution before, I have a few thoughts on that. Take a simple case of a line array [in an] infinite homogeneous ocean. Now with this concept, you put a source here—it sends out spherically spreading waves—the spherical wavefronts pass through your sensors—it gives you a set of measured pressures. You take those measured pressures and backpropagate—replace them by complex conjugates and propagate backward. It's a very standard problem in diffraction theory. Of course, you don't have this part of the wavefront—you just have this section—so when you backpropagate, this wavefront kind of gets distorted at the edge, something like this—and when you come back to the target, it will not converge to a point, it converges to a diffraction-limited focus. You have some kind of ray picture, rays something like this. You have a spread of—an ellipsoid or—actually, in this case it's a rotated doughnut-shaped thing. It's very—it's exactly analogous to having a lens here and a plane wave coming in. A plane wave coming into a simple lens, focuses to a diffraction limit. We'd like to estimate, say, the width in the range variable and the width in the cross-range. $y$ would be depth if it's a vertical array or cross-range for a horizontal array. The solution to this problem is textbook—Appendix 2 of Clay and Medwin [30]—

CLAY. Which I got from Born and Wolf.

—you got it from Born and Wolf [31]—it came originally from Debye. Well, Debye's theory says $\Delta x$—order of magnitude—is the wavelength divided by the square of the sine of the angle—the angle would be an angular aperture $\theta$. If the array length is $L$ and the range is $R$, then—I always use a small angle approximation—$L/R \approx \theta$ if the angle is small. This becomes wavelength divided by angular aperture squared,

$$\Delta x \sim \frac{\lambda}{\sin^2\theta} \approx \frac{\lambda}{\theta^2}.$$

A much easier calculation actually is $\Delta y$—the resolution in cross-range—1 divided by the sine of the angle, which is the wavelength divided by that angle.

$$\Delta y = \frac{\lambda}{\sin\theta} \approx \frac{\lambda}{\theta}.$$

These estimates. I learned yesterday, correspond to the Cramer-Rao bound, at least in order of magnitude If you replace $\theta$ by $L/R$, you get this kind of estimate for your range resolution $[\lambda/\theta^2]$ and this for your—you translate this into a bearing error—bearing resolution $[\lambda/\theta]$. The PE model -the same formulas apply—in fact, they appear in my review article on PE modelling techniques [32]—we use these formulas where $\theta$ is the—related to the limiting angle in a waveguide. We use these formulas to estimate our depth mesh—range mesh and depth mesh

Now, this is a very idealized example. If you put a surface—say a pressure-release, flat surface—these outgoing waves, they reflect off the surface, and into the receivers—you can think of as like having an image set of receivers over here where the waves—this image is also like a tilted mirror—a mirror say followed by a prism. A plane wave coming in gets tilted, goes through the lens and focuses back here. You have two paths. Then, what happens—and our pictures of numerical computation show this— instead of one ellipsoid, this breaks up into—superimposed on that are interference patterns due to basically two local plane waves—you have a set of zeros and peaks within this, and inside there's a set of much smaller ellipses, which kind of aim at the effective center of your aperture. The smaller ellipse can be estimated by and confirmed by computations. You just have a bigger angular aperture—now say of $\theta_2$—like having two paths—a bigger angle $\theta_2$ The same formulas apply now with $\theta$ replaced by $\theta_2$ In other words, your resolution increases when you get to two paths instead of one because in effect you have a greater range of angles determining the source location That can be generalized to a large number of multipaths, and the more multipaths—the largest—what finally determines your resolution in a very complicated, real ocean environment is simply the largest angle of propagation that connects the source to the receiver—not the largest but the total spread in angle. The spread in angle determines your resolution and [for the] deep ocean sound channel, you know the typical spread in angles is something like 15° or 20°—plus or minus—say if you like 20° then you put 40° here and gives you your resolution in range—or horizontal if you have a horizontal array.

How much time do I have—roughly? I don't want keep people from missing their airplane and I don't want to miss mine either.

> DICUS We should be breaking for lunch and starting at 1:30. We really only have an hour for lunch—if we're going to stay with our schedule. Since we have one more talk we should stick with that as much as possible—

So I'll try to wrap up and—I could just show the slides. Could you turn the room lights completely out? They're colour slides and they need a lot of light—to be dark. Do I control the focus?

This [Figure 4] is a typical output. It's photographed with a 35-mm camera directly off the face of a CRT—a colour monitor. It's about 512 by 512 approximately—resolution—number of pixels We plot here 3 separate fields. These are 3 computations, each one typically 20 seconds or so, so you fill up the screen like this in the order of a minute or so Usually it takes more time setting up each run than it takes to compute. This is your typical—you have seen a lot of pictures like this—the target of course is there. Now the scenario then—one large set of these pictures—is a shallow water situation—pertains to the experiment Homer did last summer off the NOSC [Naval Ocean Systems Center] tower. The water depth is 66 feet and the frequency in almost all runs is 3 kHz. The range here typically—a [*sic*] 1000

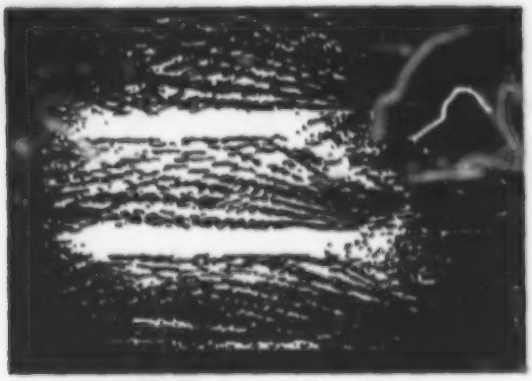

**Figure 4:** *Slide #1.*

feet or so. Now our graphics—in a sense is high quality but primitive. It takes a lot of effort to put labels on these things and unfortunately we haven't had the time or patience to do that —we meaning one of the three of us—so because there's no labels, the only way to figure out really what was being done is to go back to your laboratory notebook and check all your numbers I did bring them along, but I don't want to—to be precise about anything I'd have to refer back I believe in this case the maximum range was about 600 feet—which is about 10 times water depth All these problems scale, of course So, you could scale it to deep water except for the sound speed profiles—instead of 3 kHz it might be more like 30 Hz, and 10 water depths also scales so you're talking like a CZ or so in a deep ocean 3 kHz, in this case a vertical array, spanning the water column. [In] the PE model we compute within the bottom sediments and substrate. We display, in this case, only the water column—in other words the upper 66 feet of the computation—the bottom is flat. The sound speed was—the sound speed profile was downward refracting—we had about 10 or 15 points in it—a few small wiggles—at this frequency and range is not a dominant effect but it is important The bottom was reported to be a good reflecting sandy bottom—Homer gave us a bottom loss curve—with the PE model we have a way to translate a bottom loss curve into a geophys—a geoacoustic type bottom—with a critical angle and a loss and a density and we have a substrate And that's all in here. The environment is fairly realistic, the model we think is accurate, but we're doing simulations. The backpropagation is done with the same model as the forward propagation So we put a source—the forward propagation we put a source there with the same model, we collect the sets of pressures with a vertical array of sensors and then backpropagate it You have to get exactly the correct answer at the source The colour scale here is in dB units We do things absolute, so we have to figure out what kind of source levels we're looking for In this case we gave the target—Homer's data had no absolute numbers connected to it. In our simulations we typically just make up a number like 150 dB [re 1] $\mu$Pa for source level Then, when we backpropagate, we set our lowest colour to be 150 In this case we really know what the source level is we're looking for Each other colour represents an increment of 1 dB Actually, this darkest colour—the top of the darkest colour—the darkest colour represents numbers below 150—this would be 150 to 151 151 to 152 etc There are 15 dB between bottom and top If this were in focus we

could actually look at the—it could be our fault adjusting the focus of the camera—we look at the screen. [there are] a few pixels in there which are exactly at 150, say a half dB. The colours, say this light green, is normally down by 5 dB or so—no about 10 dB. This blue background—maybe—should be about 8–15 or so dB—close to—on the average. From that I can deduce the number of sensors. I believe in this case 48 sensors. The wavelength here is about a foot and a half, so the spacing between the sensors was about 40 to 50—about 1 foot, so roughly a half-wavelength spacing between sensors. 48 sensors—10 log of 48 is—how many dB is that?

UNKNOWN PARTICIPANT: 15 or so.

About 18–17—something—about 17—3 dB off of 20. So that correponds to this estimate. The background level is down from the peak by about 10 log $N$. And in a real sonar-type system you have to process against this kind of background, even when there is no true noise. As I was going to say, though, if you put in noise, it turns out, it also contributes background to order of $1/N$, whether—I think whether correlated or uncorrelated. In a way you can think of this background as being highly correlated noise, perfectly coherent Green's functions and pressures, it's just that they don't add up coherently—they add up incoherently—random phase, and it's as though you had uncorrelated noise contributing to a background of order $1/N$ True ambient noise added into the pressures, are the same level, $1/N$ or down from the peak by 10 log $N$. What we are doing here is keeping the source location fixed, and varying the number of sensors. Like Dick Heitmeyer and others, once you get a program like this running, you just start varying parameters and trying to teach yourself the physics of what's going on. Here we kept the spacing fixed and decreased the number. Then you see—here we're doing a true backpropagation—the array is here—so there are no fields coming out at large angles because no fields came in at large angles. Large angles are attenuated by the bottom. And you see a diffracted-limited focus at the range and depth of the target within a few wavelengths, in this case. Plus if you look more carefully, it's very clear—there's a beam of energy aiming down toward the bottom then coming up to the surface—down to the bottom—hitting the target. Another beam of energy going up, and you can kind of see the ray structure of this acoustic field, even though it's a full-wave field Very common observation of PE runs, we tend to see ray-like structures especially at the higher frequencies I don't mean 3 kHz necessarily but if you have a large number of wavelengths in the water depth, like a 100 or so, that's when you tend to see ray-like structures. You can also think of this—this being a multipath intereference between a lot of raypaths going up and down in the water column between the receiver and the source. At a particular range and depth, all these raypaths converge, certainly up here, they all converge to give you a very sharp diffraction-limited focus spot. Of course as you decrease the number of sensors, you get some spreading, as others have seen more in range than in depth, you tend to be better able to estimate depth than you can range

I have a huge number of these [Slide #2 not shown]. Every day, typically we make dozens and dozens of these We keep them in a big box of slides, and even after a month we have a thousand of these—all of them unlabelled [Laughter]

KUPERMAN That's almost useless

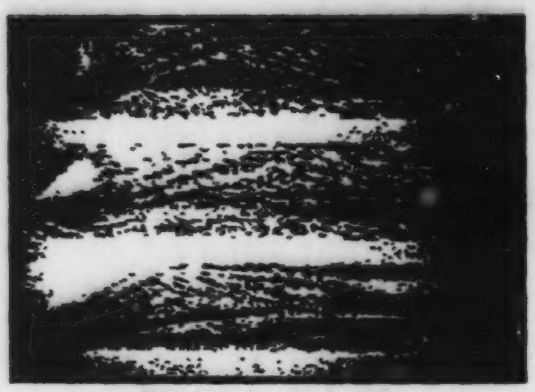

**Figure 5:** *Slide #3*

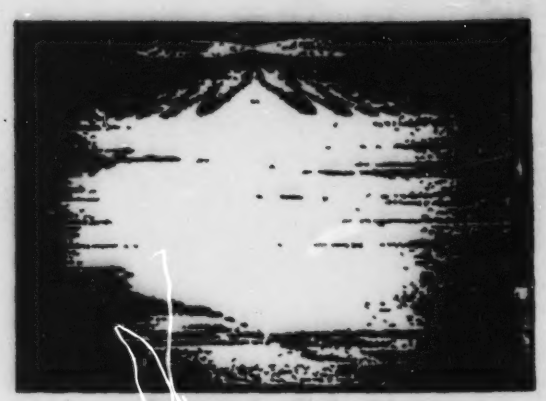

**Figure 6:** *Slide #4.*

Well. we have our lab notebooks where we carefully write down—but then we have to go back—

KUPERMAN: What are you showing?

Same kind of thing, we're just changing the number of sensors—I don't know what to—I think at the end we have 16 sensors, half a wavelength, and that gives very good localization in the ideal case—perfect knowledge of the Green's function in the ocean.

KUPERMAN: Did that picture actually change from the previous one?

It's possible but it didn't. [Shows Slide #1 again] Nope. Sometimes we make two copies of the slides in case we lose one—if we picture something particularly good—[There is some difficulty with the projector at this point] I think that inner [35-mm carousel] ring I took off If you put that inner ring back on it might hold them down.

OREST DIACHOK. If that picture didn't change I hope you made the same point twice

I would have if I knew Each time you look at these things, something different occurs to you and—education is a continuing process. There's no light on— Is that really a blank? I thought that I put in a range-dependent case

KOCH It got skipped because we were taking it in and out

Well. I'll skip the range-dependent case that I showed at the Acoustical Society meeting Of course. the model is fully range-dependent You can put hundreds of sound speed profiles in there—complicated range-dependent geoacoustic bottom with bathymetry taken from detailed nautical charts or defense mapping agency charts—the properties of the bottom can be fully range-dependent. etc., etc. We've done that. The model is reciprocal. so it's guaranteed—even in the range-dependent case—that you will get a—the correct position and the correct value for a source level—when there's no noise [and] perfect Green's functions Here [Figure 5] we're reducing aperture more and more—maybe before there was not 16—but this is about the same—16 [top]—maybe 12 [middle]— this one I'm pretty sure is

8 [bottom]—sensors—a half a wavelength apart—so $L/\lambda$—array length to wavelength—is about 4, and that was the condition of Homer's experiment. Of course, this is not the real data but simulations. The target location becomes more and more difficult to localize in range, especially as you reduce your aperture. But it corresponds still to a diffraction-limited focus from a lens that's only 4 wavelengths in diameter, with a lot of mirrors in here to reflect the waves. With practice, with training, it's not hard to identify—even without the colour scale—just from the pattern—one can identify where the target is. We have tried this out on secretaries, and janitors, and other visitors to our lab [Laughter]. They would walk up to the screen and say that's the submarine or that's it—whatever it is—even down here [bottom].

This clearly [Figure 6]—there is some interest is applying artificial intelligence concepts, expert systems analysis to this kind of thing. With training you can definitely improve your ability This is the range-dependent case. I think otherwise, [it is] pretty much the same— 3 kHz—bottom slope—this constantly sloping bottom—10°—a very strongly range-dependent case. In this case [top] we show the forward propagation—so we have a point source here—the sloping bottom is shown there—you notice that the sound wave does penetrate and refract according to Snell's Law because of the change in sound speed between the water column and the sediments. You see again these ray-like structures coming down and bending—Snell's Law And then in the bottom of the sediments—in the substrate—we add this artificial numerical absorption as always to prevent reflections. And these are waves coming out from the source, bouncing up and down, interfering—single omnidirectional—it's not quite omnidirectional because of course in PE, knowing in advance that the very steep angles will be strongly attenuated, they just come straight down to the bottom or first up and then down to the bottom, they [will] disappear from the problem at longer ranges. What we do is—don't even emit them. We just take them out of the source. So, it's an effectively omnidirectional source and this is a typical CW transmission loss plot from the PE model. Except, actually done twice— two runs—one upslope—one downslope. And then just adding them together to get this plot. In this case [middle], we had a vertical array on the upslope side—I don't remember exactly—well you can see it didn't fill the entire water column, just the upper part of it Backpropagate downslope, and right where it should be—although I don't see it—oh, white, see we changed our colour scale [Laughter]. It used to be dark red but that's kind of hard to see when the next colour is red also, so we put white there. So [a] white pixel—unfortunately the background is red—so it's hard to see. But a white pixel in there means that it got the target—source level of 150 dB. Now this is a blow-up [bottom] of the upper portion of the same backpropagation run. There's the target—easy to localize.

Now I—[Slide #5 not shown], the target's there [top], there [middle], there [bottom]. Don't ask me what's going on here—I have so many slides—I really should run through more quickly here. I think mostly aperture and number of sensor effects This [Figure 7] looks like—I know this is higher frequency—it's 6 kHz. We doubled the frequency one time, also doubling the spacing—with and without doubling the spacing—to see if it's really the aperture, whether it's the number of sensors, or whether it's the spacing between them. And our conclusion, like others' discussed at this workshop, is that it's the total aperture that counts as far as the focal spot size but as far as the localization noise background away from the target, it's the total number of sensors. The background is down $10 \log N$ and the spotsize decreases as the aperture increases

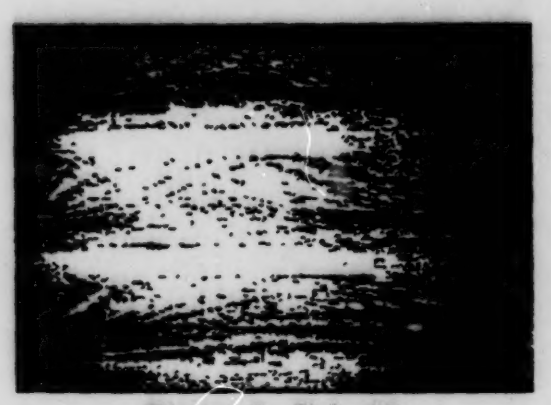

**Figure 7:** *Slide #6.*

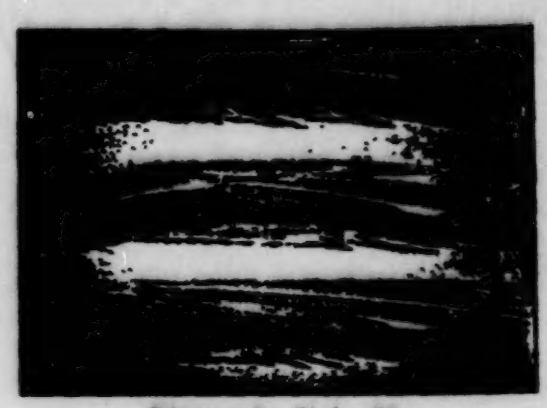

**Figure 8:** *Slide #7.*

DICUS What kind of ranges were they? Are we seeing one convergence zone, or—

No—again shallow water. downward refracting. Not very much refracting. You kind of see ray paths bouncing up and down here. So that particular raypath has bounced off the bottom once. probably twice. and into the receiver.

DICUS Shouldn't this background look very much like the ambiguity surfaces of say Dick's shallow water computations?

Yeah I think they do We have more dB levels. We're not just—10 dB means 0.1—correlation function—if you have a normalized correlation function. That's about this greenish colour When you get up toward 15–16 dB, you're down around 0 02. I would guess—

DICUS So you're showing more dynamic range than perhaps Dick was showing Is that right. Dick?

More dynamic range—

DICK HEITMEYER Yeah. I think so.

Here's a case [Figure 8] similar to the one that Dick also showed. Replace this good reflecting sandy bottom by a soft absorbing bottom. We went to the extreme limit. In the PE model we know how to make it totally, absolutely black-body absorbing So what s left—there are only the water-borne paths and the surface-reflecting paths Now there is always one surface path—very small angle—you don't have too much refraction And then there's a sort of direct path like this [top], that's slightly curved down—surface reflecting path There is nothing else Because anything that comes out—steeper angles—will hit the bottom and disappear As you would expect—effectively we are reducing the angular aperture This shallow-water waveguide now does not allow higher angles of waves to propagate—only low angles There are only a few modes. if you want to use a mode picture—a few lower modes So what happens is your resolution degrades. if you want to call it that. and since there s no noise here, you can always locate. precisely. it's just that you—it's not as sharp a focus

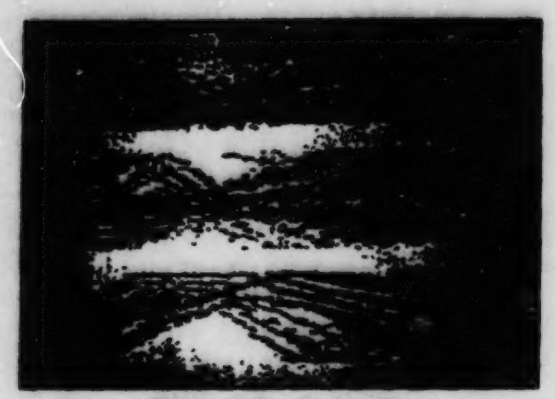

**Figure 9:** *Slide #9.*

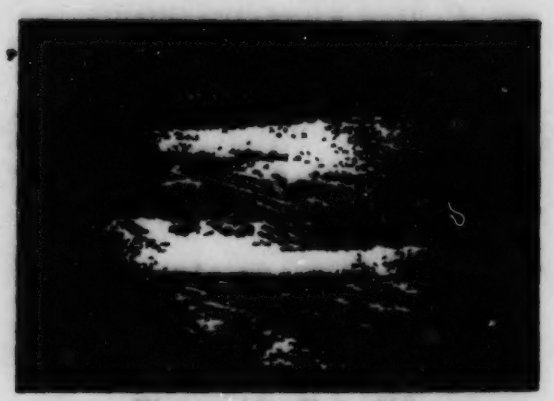

**Figure 10:** *Slide #10.*

as it is when you have higher angles and more modes And this [bottom] is a fairly small aperture—direct path and there's a surface-reflected path.

[Slide 8 not shown.] This localization is pretty much independent of target depth in shallow water—here [top] near the bottom, [middle] near the middle, [bottom] somewhat toward the surface—still many wavelengths away from the surface.

You go to other ranges [Figure 9]—shorter range, deeper target [top], about in the middle [middle], toward the top [bottom]. Again you see all this ray-like structure. We're back to the harder sandy bottom. So more eigenrays, more modes of propagation, better resolution. This break-up of the larger focal spot into many small ones is obviously due to a surface path and a bottom one The bottom path is pretty small-angle compared to the direct. It's the good surface-reflecting path that breaks up the larger ellipse into small [ones]. Target there [middle] and target there [bottom]. Here of course you get bottom-bounce paths interfering [bottom]

Still shorter range [Figure 10]. I think here we're down to something like a–175 feet as I recall. Target there [top], there [middle] and there [bottom]. This really helps [middle, high-angle path], from a pattern recognition [situation?]. Once you get that path, you know it has to be at the intersection Out here you don't see anything quite like that.

Here's a forward propagation [Figure 11] [top] from a source at that depth out in range and several backpropagate—two other back—well the same one actually—this [bottom] is a blow-up of that [middle]. So here we're only showing the upper half of the water column—all ranges. That accounts for the apparent change in pattern, but if you look, all of the structures are the same. Primitive case—aperture is about half the water depth.

DICUS. Fred, how many more do you have? We are starting to run out of time.

Ok, what I'll do is zip through them one after another, and if there are any questions [Slide 12 not shown] or comments—[Slides 13 & 14 not shown]. Simulation studies like this are just endless [Laughter]—how many different environments you want to study. We studied as many as we had time for [Slides 15 & 16 not shown] and kept the computer running for a few months

DIACHOK: Fred, did you do any mismatch studies?

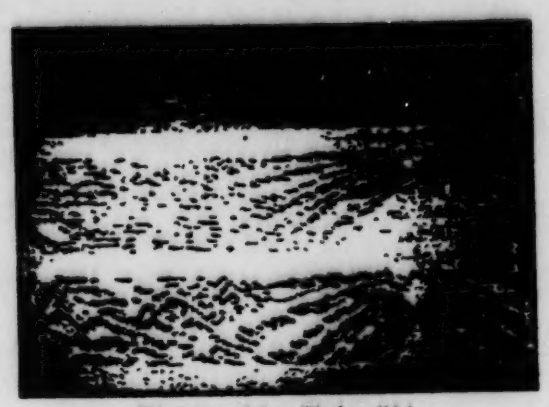

**Figure 11:** *Slide #11*

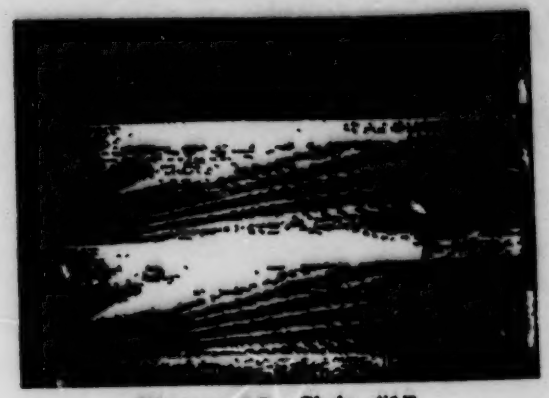

**Figure 12:** *Slide #17*

Yes We made a fast one already.

DIACHOK What did you conclude from them?

In shallow water. if you don't know anything about the sound speed profile, just iso-velocity—it doesn't work at all. You can't even iterate on that. With a simple bilinear approximation to this 15-point profile. you can do very well That means we say did a forward propagation with a 15-point real profile—did backpropagation with a bilinear profile. With a good fit, we do as well as with a real profile Then we did a case where we let some parameters—deliberately made some parameters wrong Like the bilinear thing you have 2 gradients We made one gradient way wrong And then we tried to implement what Homer suggested and maybe some others searching that set of parameters to bring in a sharp focal spot. We know that a real target—if it's actually omnidirectional—that it does behave like a point source so we should be able to get a diffraction-limited image of that target If we don't. that means something's wrong—either the environment's not right or the Green's function is not right

DIACHOK Did you do this in a random way. or do you have some iterative procedure?

No we use an iteration based on perturbation theory—the simple. linear. first-order Born approximation perturbation theory—you can get a first correction to the environment. Based on the difference between what should be there and what you actually see, in terms of spot size—spot amplitude

DIACHOK. How many runs did you have to do before it converged?

It never converges—you never get exactly 1 for the normalized correlation function And it just—what we found is that it oscillates. it doesn't seem to converge

DIACHOK So basically it just [garbled] until you get one that fits

But by eye we just --the [slow?] parameter would oscillate back and forth and we would just stop it when it looked like a good focal spot A lot of work that has to be done there. and it's a good and important area of research—by no means have we solved the problem What it amounts to -

DIACHOK: It seems to me—I've got a question again. I've got an [garbled]. There's no—you don't discern trends—approaching the right solution. You just—you try many differnt ones and one of them just happens to be right on and that works. All the others—

No, it does trend.

DIACHOK: It does?

It does trend. We found, in the cases we studied, as others said, it does not put the target in an incorrect location—mismatch as you call it. It just—the sharp focal spot just fades out. But it does come in, when you get a reasonably good estimate of the parameters. It comes into a reasonably sharp focus. Theoretically, determining the environment acoustically with coherent type processing is the problem of holography—it's acoustical holography as opposed to tomography. Where you're trying to determine a function now—sound speed as a function of depth and range—not a small parameter set describing the target. And determining a function is a hard thing to do And that what's you really want to do—not just put in a few parameters and try to estimate them. And that's what we're trying to do—we're trying to—similar to what Clarence is doing—imaging the bottom for example. We're trying to image the volume and find out what the sound speed structure really is—in the volume. And there are ways to do that, systematic, but not totally understood. Just like there are ways to image the bottom or image the surface, if you have a rough surface or ice canopy, you d like to get an image of that. And this technique, in principle, I think allows you to do that. But not totally proven and the effects of noise not totally understood.

Here's a case of small aperture [Figure 12]—very small—hard to localize. The target's is around here—

Here's a different kind of plotting format [Figure 13]. We call these things butterflies or bow ties, I don't know what others call them. [Slide 19 not shown.] This kind of plot—black is where the target is. [Slide 20 not shown.] Here you take the center of those black spots, and that is where the target really is. [Slides 21, 22 & 23 not shown.]—certainly ambiguity here

Here is multiple targets [Figure 14]. I'll just explain that this is a single target—[top] backpropagation Second [middle] separate single target. Two targets [bottom] present at the same frequency, at the same bearing, in this case the same range. Very easy to separate in this way Dick Heitmeyer said he wanted to do, and didn't. It works, no problem at all.

Here [Figure 15] is a case of two targets with different source levels—in this case [top] 150 dB $\mu$Pa—147.5—same frequency, same bearing, same range. [middle] 150, 145. [bottom] 150, 140. 10 dB down if you read your colour code. Of course, if you change, if you shift the scale up you can make this value much larger, easier to identify. This shows from a detection point of view—I guess there's no time to talk about—if you simply threshold these plots, which I think is still the right way to do it, you certainly may miss a weak target. If he sends out a decoy that masks his sound at the same frequency, at the same bearing. Put it up on a float above him—it would be pretty hard to detect, but not impossible. Normally for a threshold you get the loudest target What you can do in principle, once you know where he is and what his source level is, you go back and do a forward propagation—get the pressures due to that guy and subtract them out. That's sometimes called a cancellation

**Figure 13:** *Shde #18.*

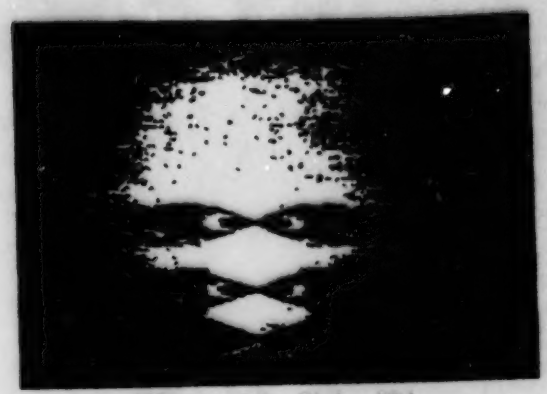

**Figure 14:** *Shde #24.*

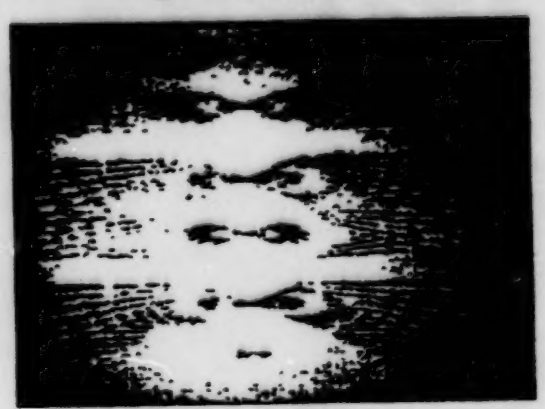

**Figure 15:** *Shde #25*

**Figure 16:** *Shde #26*

procedure In the perfect case. where you know exactly the Green's function. it works. It has to work—it can t be wrong You can cancel that guy out perfectly. then reset your threshold and very easily detect the other guy. that might be the true target

Here's a case of 3 targets [Figure 16] Here [top] there are 2 targets and a third—these targets are about 2 wavelengths apart and of course they look like a dipole—that raises the whole issue of what happens if the source is not omnidirectional Suppose you have a directional source Suppose you have a directional receiver—a DIFAR type element. We have that thing in hand. figured out at least theoretically and in simulations 2 wavelengths in this particular environment—no problem in separating them. If you get down to less than a wavelength of course there will be problems But even in principle. there is no way to tell a—2 separate monopoles from say a dipole or a single target or a single source from some other directional—directivity [middle] Here are 2 targets same depth. different range Little bit of a problem getting the true source level of the far-away target—like a baffling effect—for some reason that I don't fully understand [bottom] Here are 2 targets. different ranges. different depths The hardest case is when they are at the same range—I mean same depth but different range

DICUS Fred. can you finish up in a few minutes?

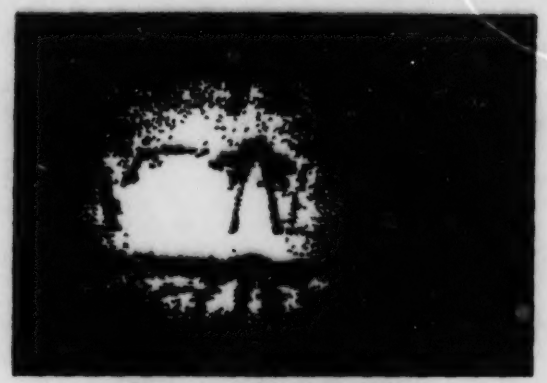

**Figure 17:** *Slide #27.*

Yes. I—a deep ocean case [Figure 17]—50 Hz—100 nautical mile range—typical Sargasso Sea sound speed profile with a double minimum. Target is there [bottom]. [Slide 28 not shown] Other case. target is there [top], there [middle], there [bottom]. No problem telling one CZ from another. CZs about 30–35 miles apart. In the ideal case, you can easily tell which CZ the target is in. if you have enough sensors. [Slide 29 not shown] Top is forward, bottom is backpropagation [Slide 30 not shown]. Here [Slide 31 not shown] is some deep ocean cases at 50 Hz with multiple targets. I'll leave it to you to say where they are [Laughter]. It's not always so easy. Here we're showing only the upper 1000 m or so.

Here [Figure 18] is a comparison between normalized and unnormalized correlation functions for $L/\lambda$ of 4—very small aperture. [top] This is unnormalized—it's a real acoustic field—it satisfies the pressure-release surface boundary condition. [bottom] The normalized correlation function is not a real acoustic field and does not satisfy the boundary condition. You can see. non-zero values at the surface. I'm sure—other people have seen that too. The real target is in here. A little bit of a problem with the unnormalized but the nearer [to the surface] is solved by the normalized correlation function. Under extreme conditions when you are limited by very small aperture. the normalized correlation function—having the Cauchy-Schwartz inequality—[is] more certain than the unnormalized correlation function.

**Figure 18:** *Slide #32*

**Figure 19:** *Slide #34.*

Again [Slide 33 not shown], multiple targets. top is forward. bottom two are back. possibly Again [Figure 19], top is forward, bottom two are back Unnormalized [middle], normalized [bottom]. You can tell from the—what happens at the surface. That's it.

# 3 Final Remarks

For completeness, the "extended abstract" that L. Nghiem-Phu, F.D. Tappert and S.C. Daubin provided to the organizers of the '85 Workshop is enclosed as an Appendix. DJT would like to thank both Fred Tappert and Lan Nghiem-Phu for their endorsement of this transcription project. Lan Nghiem-Phu provided scanned images of the 8 figures that appear in the Appendix

# Appendix

# A "Source Localization by CW Acoustic Retrogration," by L. Nghiem-Phu, F.D. Tappert and S.C. Daubin

## A.1 Introduction

A high resolution method for passive localization of a narrow band (CW) acoustic source has been developed that solves the Parabolic wave Equation (PE) for the acoustic Green's function in a "retrograted" mode.

Results obtained during an investigation of this technique of narrow band acoustic retrogration has been reported [33]. The emphasis in the first report was the use of real data to test the feasibility of source localization by acoustic retrogation in a realistic ocean environment

In this report [34]. we have included results that were obtained while investigating other issues that are also fundamentally important. In particular, we have studied the consequences of imperfect knowledge of environmental factors such as the sound speed profile, bottom characteristics etc.. and the merit of this technique when two or more coplanar acoustic sources are present. Alternate equations that describe the correlation functions are briefly introduced in Section A.2 Results of various numerical simulations both for shallow and deep ocean scenarios are presented in Section A.3. Section A.4 discusses possible improvements by use of the normalized correlation function and a summary is given in Section A.5.

## A.2 Narrow Band Acoustic Retrogation

The technique we call narrow band acoustic retrogation can be used to determine the unknown location of an omnidirectional acoustic source in a range-depth plane $\vec{x}_s = x(r_s, z_s)$. The acoustic source is assumed to be radiating acoustic energy which is narrowly confined around center frequency $\omega$. We have assumed that the bearing is known so that the source location is determined by its range $r_s$ and depth $z_s$ in the corresponding range-depth plane.

The radiated field satisfies the reduced acoustic wave equation

$$\nabla^2 p_s + k^2(\vec{x}) p_s = -4\pi p_{s0} R_0 \delta^2(\vec{x} - \vec{x}_s). \tag{A.1}$$

where $p_{s0}$ is the reference sound pressure at a reference distance $R_0$. We take $R_0$ to be 1 yard The pressure ratio. or Green's function can be defined as

$$g_s(\vec{x}) = p_s(\vec{x})/p_{s0}. \tag{A.2}$$

At $N$ omnidirectional receivers located at $\vec{x}_n$, $n = 1, 2, \ldots, N$, the received pressures due to the source at $\vec{x}_s$ are

$$p_n = p_s(\vec{x}_n) = g_s(\vec{x}_n) p_{s0}. \tag{A.3}$$

The Green's function due to a source at $\vec{x}_n$ satisfies

$$\nabla^2 g_n + k^2(\vec{x}) g_n = -4\pi R_0 \delta^2(\vec{x} - \vec{x}_n). \tag{A.4}$$

By the principle of reciprocity,

$$g_n(\vec{x}_s) = g_s(\vec{x}_n). \tag{A.5}$$

The source localization problem may be simply stated as: given the received pressures $p_n$, $n = 1, 2, \ldots, N$; it is required to determine $\vec{x}_s = \vec{x}(r_s, z_s)$ and $p_{s0}$

Our solution method is based on the numerical construction of a hypothetical field $h(\vec{x})$ which has the property that $|h(\vec{x})|^2$ has a local extremum at $\vec{x} = \vec{x}_s$; and also the $p_{s0}$ can be determined from $|h(\vec{x})|^2$. This method is essentially a correlation method, similar to the "matched field" method of Bucker [16], Devaney's "back propagation" method [35], and Heitmeyer *et al.*'s [22] ambiguity function. We have focused on two alternate fields defined as

$$h_1(\vec{x}) = \frac{\sum_{n=1}^{N} p_n^* g_n(\vec{x})}{\sum_{n=1}^{N} |p_n|^2}, \tag{A 6}$$

and

$$h_2(\vec{x}) = \frac{\sum_{n=1}^{N} p_n^* g_n(\vec{x})}{\sqrt{\left(\sum_{n=1}^{N} |p_n|^2\right)\left(\sum_{n=1}^{N} |g_n|^2\right)}}. \tag{A.7}$$

where the symbol * denotes complex conjugate quantity

Equations (A.6) and (A 7) correspond to methods A.2 and A.3 mentioned in our earlier report [33] Both can be interpreted as correlation functions between the actual received acoustic pressures $p_n$ and the pressures that would be received if the source were situated at location $\vec{x}$. Equation (A 7) is different in that it represents a normalized correlation function that is absolutely bounded by unity. In this report, we present displays of pseudo source level (PSL) functions defined as

$$\mathrm{PSL}(\vec{x}) = -10 \log(p_r^2 \cdot |h(\vec{x})|^2). \tag{A 8}$$

where $p_r = 1\ \mu$Pa is the reference pressure. It can be shown that the source level SL in units of dB re $\mu$Pa re 1 yd is given by

$$\mathrm{PSL}(\vec{x}_s) = \mathrm{SL} \tag{A.9}$$

The computation of $h_1(\vec{x})$ is straightforward and can be done with one single computer simulation similar to a single frequency forward propagation using the PE split-step Fourier algorithm The computation of $h_2(\vec{x})$ requires $N$ computations of $g_n(\vec{x})$ in the retrogation mode thus the computation effort increases as $N$

## A.3 Retrogation Modeling

In this section we only report results obtained from the application of our retrogation technique with simulated (synthesized) pressure data The scope of our present work precludes an in-depth investigation using real data although we have obtained two sets of high quality data for shallow and deep ocean scenarios; and have done some limited data processing [36 37]

Most of our modeling work was performed using the field $h_1(\vec{x})$ of Equation (A 6) because it is more time consuming to use Equation (A 7). However, we have compared the two methods and an example is shown in section A.4.

### A.3.1 Displays of results

In general, our retrogation technique is based on a forward propagation model that generates the prediction of the acoustic pressure field in a range-depth plane. This pressure field can easily be converted to transmission loss (TL) in units of dB referenced to the intensity at a unit distance (we usually use $R_0 = 1$ yd). This TL field can be displayed in color scale for visual inspection. In all of the figures that follow, however, the forward fields, whenever presented, are not in the familiar TL units. Rather, they are presented in arbitrary units and are primarily shown in this report to provide the reader with a feel of how the acoustic energy might be distributed in a vertical section of the ocean where an array of acoustic sensors would intercept them.

The retrogated fields are color coded values of pseudo source level as defined in Equation (A.8). When the $h(\vec{x})$ field of Equation (A.6) or Equation (A.7) has a local extremum, suggesting a localized target, the PSL field displays a corresponding local minimum. This local minimum is generally coded with the color that is at the low end of the color scale (dark red or white). From Equation (A.9), the source level SL can be obtained simply by reading off the color scale. By visual inspection of this field, assessment about the success or failure of the retrogation technique may be made.

### A.3.2 Effects of environment data

One of the first questions that comes to mind is: what is the effect that the environmental knowledge (or lack of it) bears on the retrogation method?

We have attempted to address this issue in a limited way by using only one scenario. This scenario is the same that we have used in our earlier report, namely, that of the NOSC tower experiment from which processed data were graciously made available to us by Dr. H. Bucker [36] For this study, only one set of environmental data was used and the corresponding sound speed profile is given in Table 1. The experiment conducted by Dr. H. Bucker took place in shallow water having a nominal depth of about 66 feet over a sandy bottom which is a good

| Depth (ft) | Speed (ft/sec) |
|---|---|
| 0.0 | 4973.8 |
| 13.8 | 4963.9 |
| 23 0 | 4950.8 |
| 29.9 | 4944.2 |
| 36.1 | 4934.4 |
| 39.0 | 4924 5 |
| 42 0 | 4921 3 |
| 57.1 | 4921.3 |
| 147.6 | 4921.3 |

**Table 1:** *Sound Speed Profile*

reflector with critical angle of about 30 degrees. The frequency was 3 kHz and a vertical array with 8 elements spaced about 1 foot apart was used. The aperture was thus 7 feet and

it contains about 4 acoustic wave lengths while the deepest sensor was about 10 feet above the bottom.

By a routine simulation of forward propagation, we have predicted the pressures received at a vertical array due to an omnidirectional source emitting 3-kHz acoustic energy at a known location ($r_s = 607$ feet, $z_s = 16$ feet). To facilitate the illustrations, we have used an aperture that is larger than that [*sic*] was used in the actual experiment. From the results of our previous investigation, 48 sensors would be more than adequate to provide unambiguous localization. Since we are attempting to isolate the effect of environmental issues, it is better to use a large aperture to reduce retrogation noise. Thus, our synthetic array is placed between the depths of 8 feet to 55 feet.

Figure 20 represents display of fields generated by our retrogation model. Figure 20–a[2] is the field produced in the forward mode that illustrates the complicated interference patterns due to the presence of multipath conditions. Figure 20–b is the retrogated PSL field obtained using the 48 pressures predicted by the forward simulation. Unambiguous localization is clear. Figure 20–c is a "close up" of the upper half of the ocean that shows finer details.

Figure 21 represents the retrogated PSL fields that would be obtained when the sound speed profile has been deliberately perturbed in a known fashion. In Figure 21–a and b, the sound speed profile is modified only at one depth. Respectively, the sound speed at depth 39 feet was changed from 4924.5 ft/sec to 4934.5 ft/sec and then to 4947.5 ft/sec. We note that these changes are large but realistic. No apparent degradation was found and localization is clearly unaffected. When we adopt the commonly occured assumption that the medium can be represented by one iso-speed layer, then the localization process broke down completely, as shown in Figure 21–c. Other local perturbations that we used (e.g., the sound speed at the surface, bilinear representation of sound speed profile) also did not degrade the localization process.

We conclude that our retrogation technique, although quite insensitive to local uncertainties in sound speed, is critically dependent on the gross overall structure of the sound speed profile, at least for the case we have analyzed.

The environment issue is an important issue that needs further analysis; the above limited example does not pretend to be complete in any way.

### A.3.3 Retrogation with multiple targets and noise

In this section, we present some of the results obtained when two or more sources are present (at this stage, we assume that all the sources and the receiver vertical array are coplanar).

The theory of retrogation with multiple targets and noise is complicated and is far beyond the scope of this work. However, it is useful and instructive to consider some simple cases where certain simple assumptions can be made. One such assumption is that the received pressures due to spatially distinct targets can be considered uncorrelated and additive. Another assumption is that noise is not present at any of the receivers, an assumption that certainly can not be true in any real environment (but synthetically easy). One con-

[2]Here a, b and c are used to denote the upper, middle and lower panel of each figure, respectively.

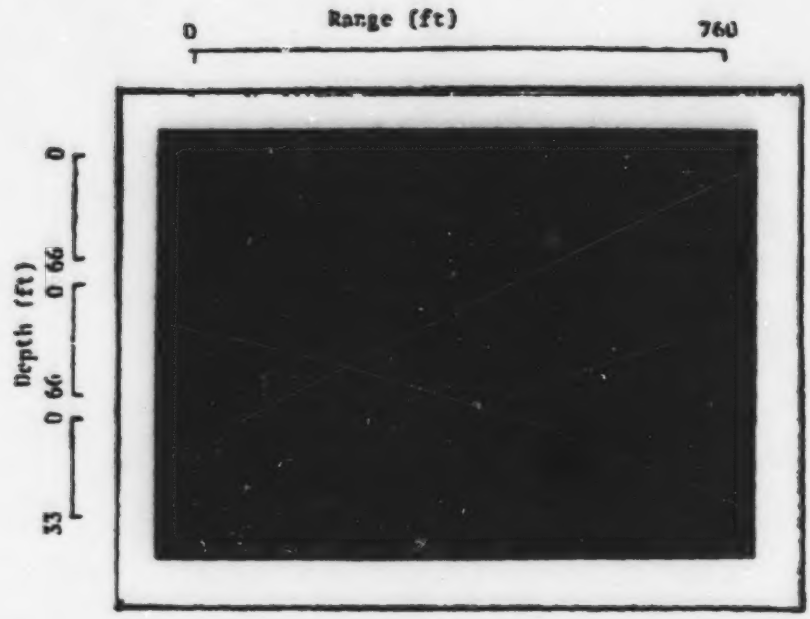


**Figure 20:** *Forward propagation and retrogated PSL fields with 48 sensors: a—forward propagation. b—retrogation; c—upper half of b.*

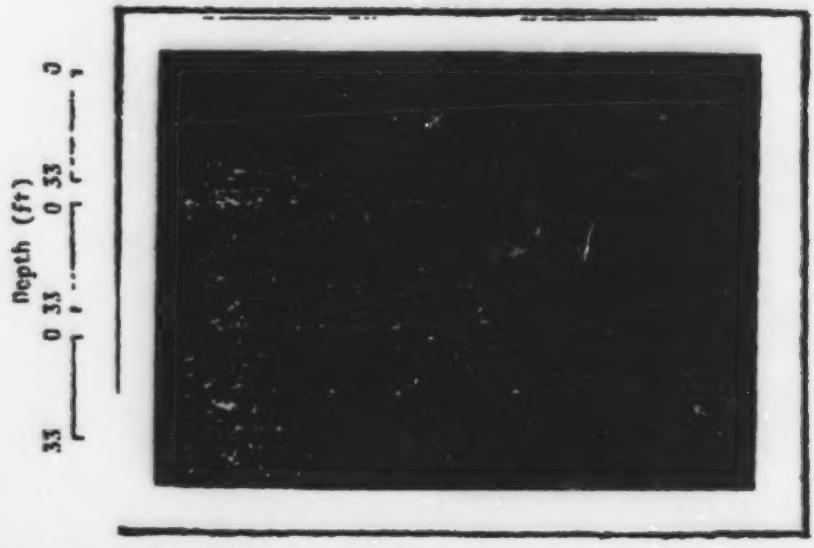


**Figure 21:** *Retrogated PSL fields with sound speed uncertainties: a—single point perturbation of 10 ft/s. b—single point perturbation of 25 ft/s; c—isospeed profile.*

sequence of noise is that the retrogated PSL field must be threshold [*sic*] against a certain "background" noise level that depends on the number of sensors and their spacings

Despite the unrealistic assumption of absence of noise in the received pressure, the retrogation process has its own background noise level from which ambiguities may arise In the following examples, we have simulated 64 sensors so that the retrogated background noise is small This is an experience that we have gained from our limited involvement with this technique.

Figures 22–24 are retrogated PSL fields for various scenarios in the same environment as that of Figure 20–a to c. The total range is now 350 feet and the total depth of 66 feet is displayed The 64 sensors practically cover the entire water depth with their 1-foot spacings Figure 22–a and b reveal the unambiguous locations of 2 single sources of 150 dB source level at the range of 175 feet at depths of 15 feet and 50 feet respectively. Figure 22–c is obtained when the two sources are present simultaneously. We have simulated this situation by coherently adding the individual synthetic pressures together at the 64 sensors It is clear that the localization of both sources and determination of their equal level of 150 dB are not degraded although the retrogated background noise has risen somewhat

Figure 23–a to c show the results when the source levels differ by 2.5, 5 and 10 dB respectively From these figures, it can be anticipated that one should start to worry about the effect of retrogation background on the localization of the more quiet source. Figure 24–a contains 3 sources of equal source levels of 150 dB at depths of 15, 20 and 50 feet. It can be seen that the vertical resolution of this array is adequate to separate the two shallower sources Figure 24–b contains 2 sources at the same depth of 15 feet but at ranges of about 91 and 182 feet Figure 24–c reveals two sources at ranges 91 and 175 feet and depths of 15 and 50 feet

It appears that localization remains adequate Nevertheless, more theoretical foundations must be developed so that the effects of noise (both from natural processes and from the retrogation technique) can be included and that the ability to localize and determine separate targets with different source levels may be assessed more realistically

### A.3.4 Retrogation in deep water

In typical deep ocean propagation conditions where convergence zones (CZs) are prevalent it is anticipated that localization by the retrogation technique can further be complicated by possible retrogation peaks in the vicinity of CZs. We have simulated such a scenario for completeness The sound speed profile we used is typical of the Sargasso Sea where a secondary sound channel is present at a depth of about 750 feet. We have neglected all reflections from a flat bottom at a depth of about 18,000 feet. Simulated pressures from a 50-Hz CW source are computed at a hypothetical vertical array consisting of 64 receivers spanning the depths between 500 to about 15,000 feet.

In Figures 25 and 26 only the upper half of the ocean is displayed Figure 25–a represents the field due to an omnidirectional source situated at a depth of 2,000 feet. The maximum range is 100 nautical miles The synthetic received pressures are obtained at the range of 50 nautical miles. Figure 25–b and c are retrogated PSL fields for a source at the depths of 2,000 and 500 feet, respectively. It can be seen that the CZ structure is clearly illustrated in the retrogated field In this ideal situation where noise is not considered, the retrogation

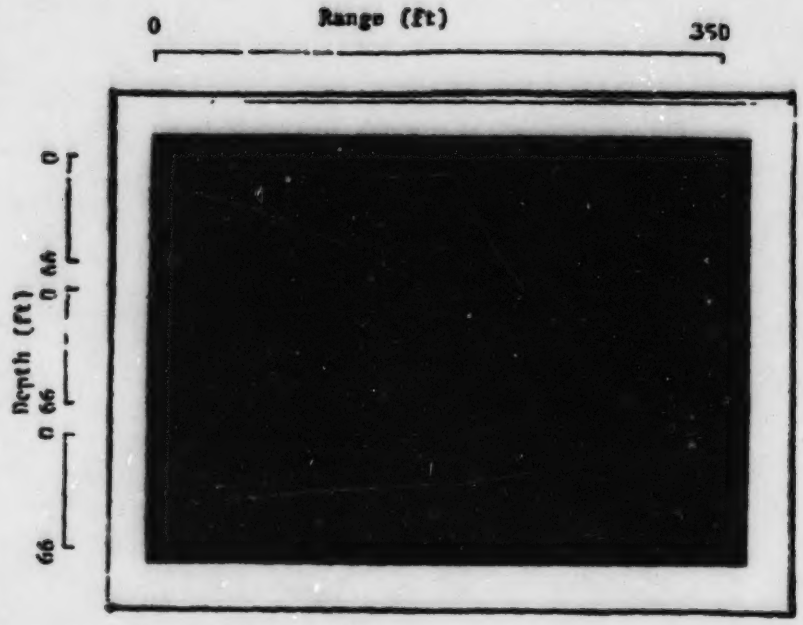


**Figure 22:** *Retrogated PSL fields with 64 sensors with one and two sources with same level: a—source depth 15 ft; b—source depth 50 ft; c—two sources.*

**Figure 23:** *Retrogated PSL fields with 64 sensors, two sources: a—SL1 is 150 dB, SL2 is 147 5 dB; b—SL1 is 150 dB, SL2 is 145 dB; c—SL1 is 150 dB, SL2 is 140 dB*

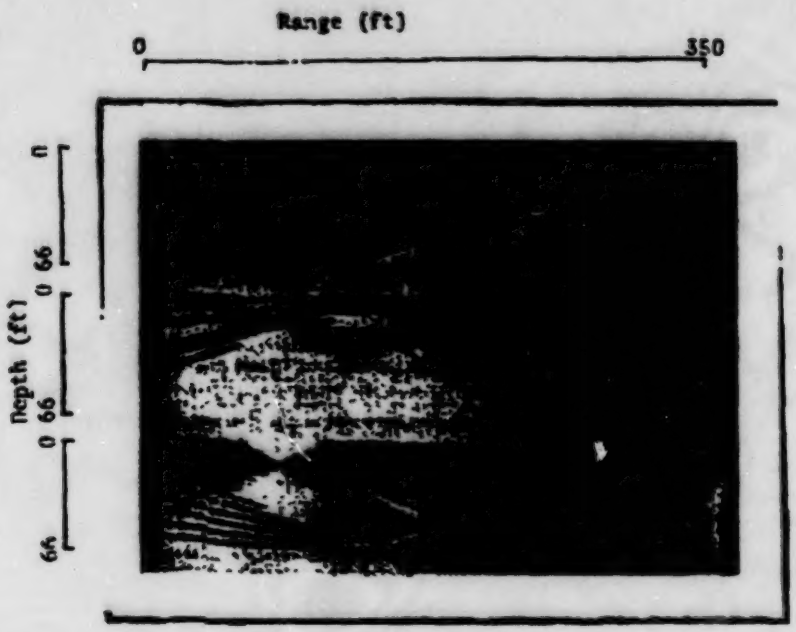


**Figure 24:** *Retrogated PSL fields with 64 sensors with two and three sources with same level a—source depths 15, 20 and 50 ft. source range 175 ft. b—source depth 15 ft and source ranges 90 and 182 ft: c—source depths 15 and 50 ft, source ranges 90 and 175 ft*

method provides unambiguous localization for both cases. A multiple sources example is shown in Figure 26–c where two sources as revealed in Figure 26–a and b are combined The range and depths are 50. 75 nautical miles, and 1,000 and 2,000 feet It is interesting to note that for a shallow source (about 5 wave lengths from the surface. as in Figure 25–c). and for multiple sources (Figure 26–c). retrogation noise level is only about $2-3$ dB away from that of local extrema. We believe that in these instances. Equation (A 7) might be more effective although we have not attempted further verification

## A.4 Comparison of Simple Correlation and Normalized Correlation Methods

In this final section, we discuss results obtained when both Equations (A 6) and (A 7) are used For this example, we return to the scenario used in Section *A* 3.2, which is similar to that used in our first report. except that we now use a source depth of 26 feet

To facilitate the comparison. we have modified the color scale to introduce the color white to code the highest intensity (Figure 27–a) and lowest PSL in the retrogated fields (Figure 27–b and Figure 27 -c) In Figure 27–a through Figure 27–c. only the upper half of the field is shown Figure 27–a is a color contour of the field predicted by a forward simulation of a 3-kHz omnidirectional source radiating acoustic energy away from its geometric center (white spot) The total range is now 350 feet while the receiver array is placed at a distance of 175

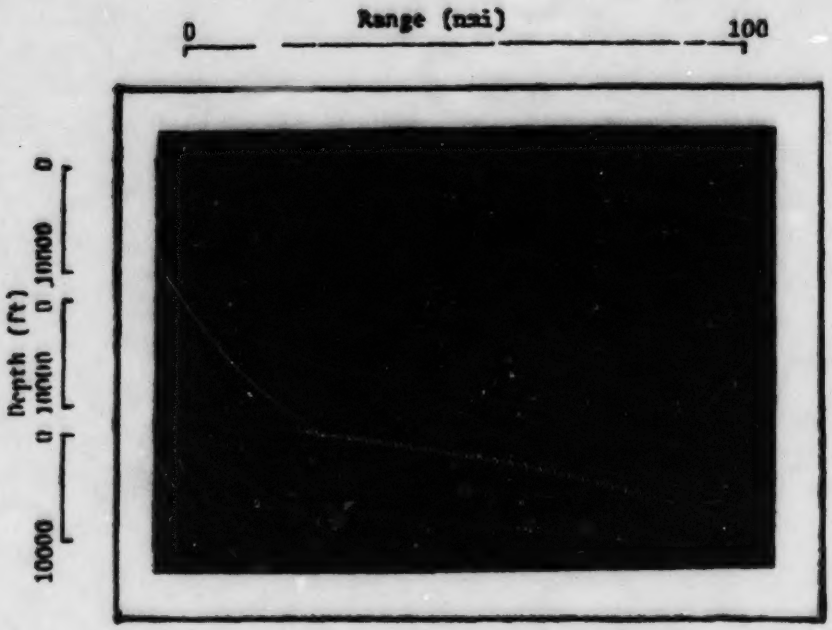


**Figure 25:** *Forward propagation and retrogated PSL fields in deep ocean: a—forward propagation. b—source depth* 2,000 *ft, range* 50 *nmi; c—source depth* 500 *ft, range* 50 *nmi.*

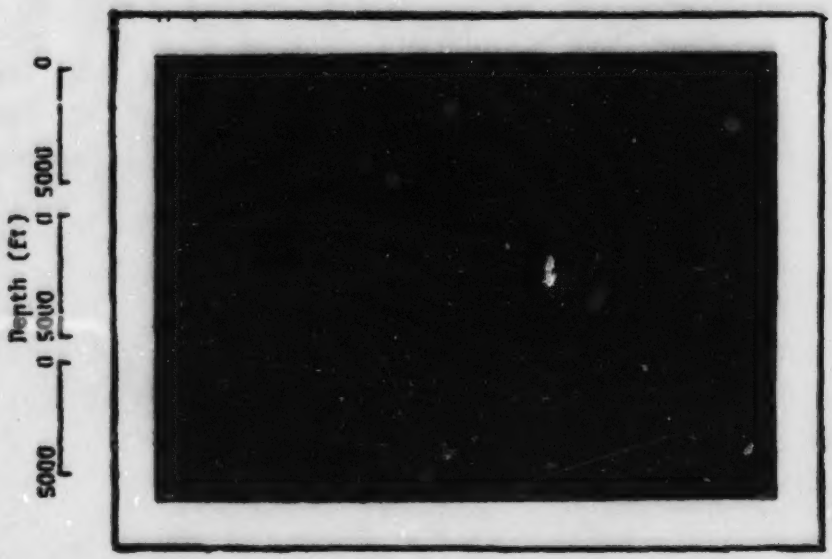


**Figure 26:** *Retrogated PSL fields with one and two sources: a—source depth* 1,000 *ft, range* 50 *nmi, b—source depth* 2,000 *ft, range* 75 *nmi; c—both sources at same level of* 150 *dB.*

feet from the source. We used 8 sensors spaced 1 foot apart to simulate a very unfavorable condition that corresponds closely to the experimental conditions. Again, it is emphasized that we only discuss synthetic pressures. The purpose of this section is to compare relative merits to two different techniques of retrogation processing

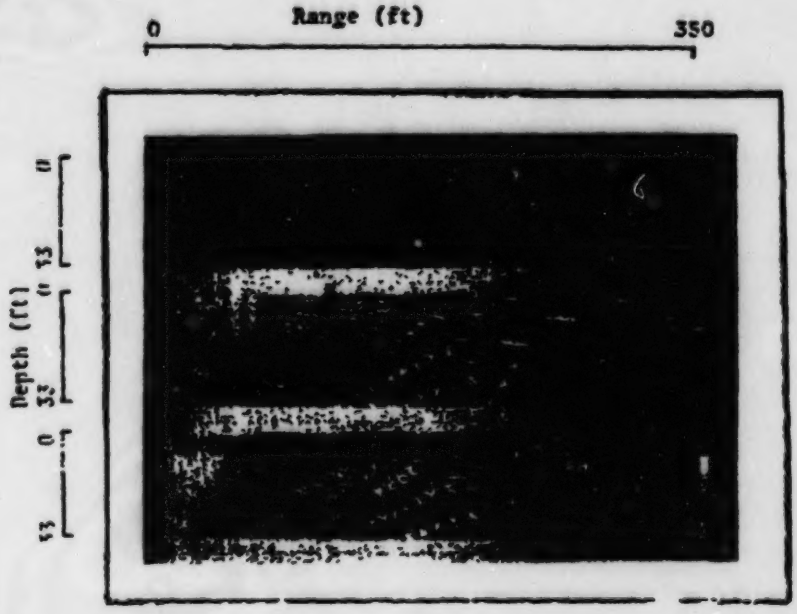


**Figure 27:** *Forward propagation and comparison between normalized and unormalized "correlation" processing. a—forward propagation; b—unormalized PSL field, c—normalized PSL field*

Figure 27–b and Figure 27–c show the PSL fields obtained using Equation (A 6) and Equation (A 7) respectively. They are similar to Figure 21 of our first report. As can be seen in Figure 27–b, the location of the source position is ambiguous We believe that this is mainly due to the limited aperture being used. In Figure 27–c, when the normalized correlation is used instead, localization is greatly improved. The level between neighbouring colors is 1 dB, thus, we have achieved a 1 dB or better gain with Equation (A 7) In terms of the computing time, Figure 27–c is obtained from 8 separate retrogations Due to another limitation of our hardware related to storage capability, it was also necessary to repeat this computation 4 times to search to the same range as in Figure 27–b This resulted in an increase in computing time by a factor of 32! In principle, when storage is not a limitation, the processing time increases as the number $N$ of sensors in use.

## A.5 Conclusions

We have shown that our retrogation technique can be useful in the determination of unknown source location in the range-depth plane for both shallow and deep ocean environ-

ments. For deep ocean scenarios, localization does not have to rely on bottom interacting energy, presumably because of the large number of multipaths that are already present due to the relatively long range and due to propagating conditions (e.g., sofar channel).

We have investigated the merit of using a normalized correlation technique using Equation (A.7). This method appears to provide improvement on the localization problem when the aperture is known to be marginally adequate for unambiguous localization. However, other simulations not presented in this report have provided us with evidence that this improvement might not be appreciable when the aperture is already adequate to obtain reliable localization by use of Equation (A.6). This suggests that Equation (A.7) should only be used in cases where every dB must be gained in aperture limited conditions. This may turn out to be what would be required when noise and incomplete knowledge of the environment are taken into account.

Further investigations are clearly needed to provide a better understanding of the power and shortcomings of this apparently promising technique of localization.

# References


1. R.G. Fizell, *Proceedings of Workshop of Acoustic Matched-Field Processing, November 20 & 21, 1985. Naval Research Laboratory, Washington, DC*, Unpublished manuscript, 1985.

2. A Tolstoy. *Matched Field Processing for Underwater Acoustics* (World Scientific, Singapore. 1993)

3. S.M. Bates and B.J. Bates. "Source localization by inversion of the parabolic equation method." J. Acoust. Soc. Am. Suppl. 1 **85**, S18. (1989).

4. R.B Evans. W.M Carey, G. Botseas and J.A. Davis, "Efficient simulation of the vertical structure of noise using the parabolic equation," J. Acoust. Soc. Am. Suppl. 1 **85**. S126 (1989).

5 G R. Ebbeson D.J. Thomson and B.H. Maranda, "Spectral decomposition of underwater sound received on a vertical line array," in *Proceedings of IEEE Oceans '91*, Vol. 3. Honolulu. HA, 1336–1343, 1991.

6. D.J. Thomson and G.R. Ebbeson, "A propagating field approach to modal decomposition and source localization," in *Proceedings of IEEE Oceans '93*, Vol. 3. Victoria, B.C., 91–96. 1993.

7. D.J. Thomson and G.R. Ebbeson, "A PE-based approach to modal decomposition and source localization." J. Comp. Acoust. **2**, 231–250 (1994).

8 D J. Thomson. G.R Ebbeson and B.H. Maranda. "A matched-field backpropagation algorithm for source localization," in *Proceedings of MTS/IEEE Oceans '97*, Halifax. N.S . 602–607. 1997.

9 D J. Thomson, G R. Ebbeson and B.H. Maranda, "A PE-based backpropagation algorithm for matched-field processing," in *Proceedings of 16th ICA and 135th ASA*. Seattle, WA. 649–650, 1998

10 L. Nghiem-Phu, S.C. Daubin and F.D. Tappert, "A high-speed, compact, and interactive Parabolic Equation SOlution GENerator," J. Acoust. Soc. Am. Suppl 1 **75**. S26 (1984).

11. F.D Tappert, "Computation intensive propagation modeling using the PESOGEN system." J Acoust. Soc. Am Suppl 1 **76**. S37 (1984).

12 F.D. Tappert, L Nghiem-Phu and S.C. Daubin, "Source localization using the PE method." J Acoust. Soc Am. Suppl. 1 **78**. S30 (1985)

13. F D. Tappert and L. Nghiem-Phu, "Modeling of pulse response functions of bottom interacting sound using the parabolic equation method," in *Ocean Seismo Acoustics*. edited by T Akal and J. Berkson (Plenum, New York, 1986), pp 129–137

14 L Nghiem-Phu and F. Tappert, "Modeling of reciprocity in the time domain using the parabolic equation method." J. Acoust. Soc. Am **78**, 164–171 (1985).

15 A Parvulescu and C.S. Clay, "Reproducibility of signal transmissions in the ocean." Radio Electron. Eng **29**. 223–228 (1965)

16 H P. Bucker. "Use of calculated sound fields and matched-field detection to locate sound sources in shallow water." J Acoust. Soc. Am **59**. 368–373 (1976)

17. C.S Clay. "Use of arrays for acoustic transmission in a noisy ocean." Rev Geophys **4**. 475-507 (1966)

18 A B Baggeroer. "Sonar Signal Processing," in *Applications of Digital Signal Processing*. edited by A V. Oppenheim (Prentice-Hall, Englewood Cliffs, 1978), pp. 331–437.

19 E C Shang, C.S. Clay and Y.Y. Wang. "Passive harmonic source ranging in waveguides by using mode filter." J. Acoust Soc Am **82** 1736–1745 (1987)

20 T C Yang. "A method of range and depth estimation by modal decomposition. J Acoust Soc Am **78**. 172–175 (1985)

21 G R Wilson, R A. Koch and P.J Vidmar, "Matched mode localization." J Acoust Soc Am **84**. 310–320 (1988)

22 R M Heitmeyer, W.B Mosley and R.G. Fizell. "Full field ambiguity function processing in a complex, shallow-water environment." in *High Resolution Signal Processing in Underwater Acoustics*. edited by R A. Wagstaff and A.B Baggeroer (NORDA. NSTL. MS. 1983). pp. 171–189

23 R Z Fizell "Application of high-resolution processing to range and depth estimation using ambiguity function methods " J Acoust. Soc. Am **82**. 606–613 (1987)

24 M B. Porter, R.L. Dicus and R. G. Fizell. "Simulations of matched-field processing in a deep-water Pacific environment," IEEE J. Oceanic Eng **OE-12**, 173–181 (1987).

25. C.S. Clay. "Waveguides, arrays, and filters," Geophys. **31**, 501–505 (1966).

26. M.J. Hinich, "Maximum likelihood estimation of the position of a radiating source in a waveguide." J. Acoust. Soc. Am. **66**, 480–483 (1979).

27. A.B. Baggeroer, W.A. Kuperman and H. Schmidt, "Matched field processing: Source localization in correlated noise as an optimum parameter estimation problem," J. Acoust. Soc. Am. **83**, 571–587 (1988).

28. H.P. Bucker. "Cross-sensor beam forming with a sparse array," J. Acoust. Soc. Am. **61**, 494–498 (1977).

29. H.P. Bucker. "Beamforming a towed line array of unknown shape," J. Acoust. Soc. Am. **63**. 1451–1454 (1978).

30 C.S Clay and H. Medwin, *Acoustical Holography: Principles and Applications* (Wiley. New York. 1977). pp. 408–414.

31 M Born and E. Wolf. *Principles of Optics* (Pergamon. Oxford, 1965).

32 F.D Tappert. "The parabolic approximation method," in *Wave Propagation and Underwater Acoustics* edited by J.B. Keller and J.S. Papadakis (Springer, New York, 1977). pp. 224–287

33 S.C. Daubin, L Nghiem-Phu and F.D. Tappert, "Source localization by CW retrogation using real data." DSC TR 03–85, July 1985.

34. L Nghiem-Phu. F.D. Tappert and S.C. Daubin, "Source localization by CW retrogation." DSC TR 05–85. October 1985.

35 A.J Devaney. "Inverse source and scattering problems in ultrasonics," IEEE Trans. **SU–30**. 355-364 (1983).

36. H.P Bucker and P. Schey, Private communication, 1985.

37. D J. Ramsdale, Private communication, 1985

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Le présent document est une transcription littérale d'un exposé enregistré sur bande magnétoscopique présenté par Fred Tappert lors de l'atelier sur la localisation des sources acoustiques qui s'est tenu les 20 et 21 novembre 1985 au Naval Research Laboratory à Washington D.C. Les questions et commentaires des participants y sont inclus. Certaines équations ont été légèrement modifiées pour tenir compte de la dynamique du tableau noir. Plusieurs des diapositives de 35 mm projetées à la fin de l'exposé ne figurent pas dans le texte, là où il en est fait mention. Une liste des références citées durant l'exposé est incluse. Enfin, un résumé additionnel fourni par les auteurs aux organisateurs figure en annexe.

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